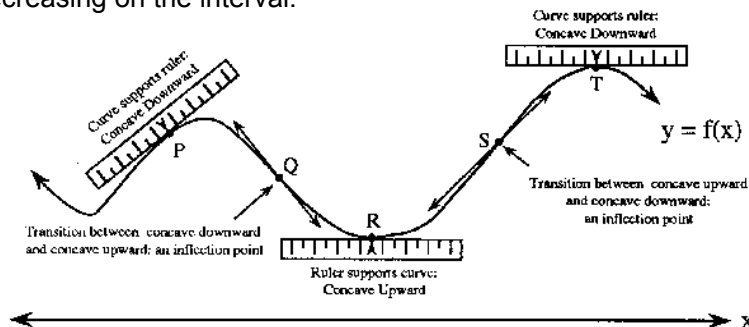


Date: \_\_\_\_\_

#### 4.4 Concavity and Points of Inflection

**Concavity** refers to the rate of change of the **tangent slope**. A graph is **concave up** on an interval if the tangent slopes are increasing on that interval. A graph is **concave down** on an interval if the tangent slopes are decreasing on the interval.



**Points of Inflection** are the points on the graph where concavity changes. Sometimes they are obvious (like in a cubic function), and sometimes they are not (see the diagram above).

When we are dealing with concavity, we are concerned with finding **points of inflection**, as well as with identifying the intervals on which a function is **concave up** and **concave down**.

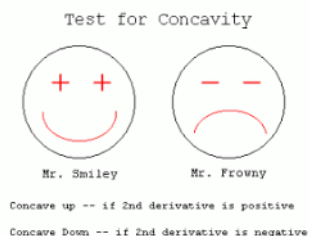
How can we find the rate of change of the tangent slope?

What are we looking for to decide if the function is concave up or concave down for a given value of  $x$ ?

How can this help us to classify critical points as maxima, minima, or neither?

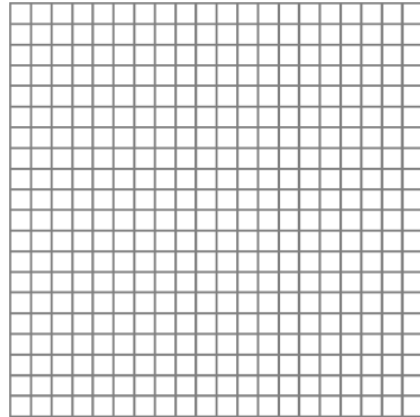
**\*This is called the second derivative test, and is usually more efficient than the first derivative test to classify critical points!**

How can we find the coordinates of the points of inflection for a function?



Example 1:

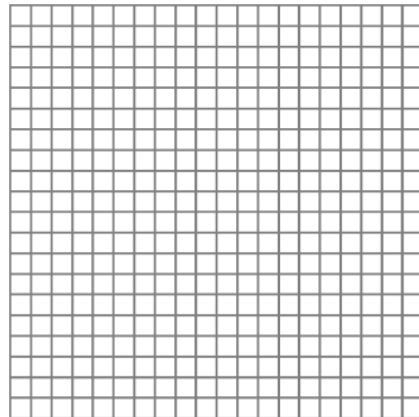
Determine any critical points and points of inflection for the graph of  $f(x) = x^3 + 2x^2 - 4x + 3$ . Use the second derivative test to classify the critical points, and state the intervals on which the graph is concave up and concave down. Sketch the graph.



Example 2:

Sketch the graph of the function  $g(x) = x^{\frac{1}{3}}$ .

|          |  |  |  |
|----------|--|--|--|
| Interval |  |  |  |
| $g''(x)$ |  |  |  |
| $g(x)$   |  |  |  |



The use of the second derivative allows us to refine our approach to curve sketching, as we can now use the second derivative to classify critical points and identify points of inflection.

Example 3:

Use the appropriate steps for curve sketching to accurately sketch the graph of  $f(x) = \frac{4x^2 - 3}{x^3}$ .

Use  $f(x)$  to identify asymptotes and intercepts.

Find  $f'(x)$  and  $f''(x)$  to identify critical points, intervals of increase, decrease, and concavity.

