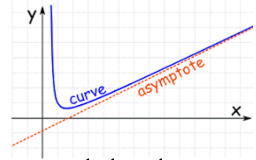
Date: ______________

4.3 Vertical and Horizontal Asymptotes



Once again, we can build on what we already know about asymptotes to make connections to limits and accurately sketch graphs of rational functions.

From Advanced Functions:

When you have a function of the form $f(x) = \dfrac{ax+b}{cx+d}$, how can you identify the equations of the asymptotes?

When will a rational function of the form $g(x) = \dfrac{p(x)}{q(x)}$ have a:

a) vertical asymptote?

b) horizontal asymptote?

c) oblique asymptote?

Using Calculus to Explain Why!

1) Limits at Infinity to Find Horizontal Asymptotes

To find equations of horizontal asymptotes, you will need to be able to consider the limit as a function approaches positive and a negative infinity. This is very similar to end behaviours, but allows you to find equations for unfamiliar functions.

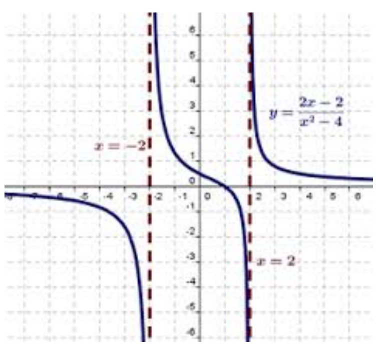
Before we can do that, let's think about the limit of the reciprocal function, f(x) = $^1/_x$, as x approaches infinity in each direction. This definition is important to find equations of more complex functions!

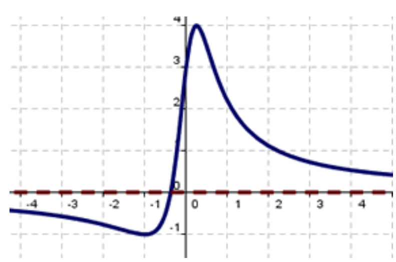
Now we can use this idea and our factoring skills to rewrite polynomial functions and use limits to find the equation of a horizontal asymptote. To do this, you will common factor the highest degree term out of the expression, creating reciprocal terms (this is sort of similar to the first step of partial factoring or completing the square).

ex/ Write $g(x) = \dfrac{2x-5}{4x+1}$ and $f(x) = \dfrac{2x^2+3}{3x^2-x+4}$ in factored form, and then evaluate the limit as x approaches positive and negative infinity to find the equation of the horizontal asymptote.

An important note about horizontal asymptotes:

A graph approaches a horizontal asymptote as x gets infinitely large in the positive or negative direction. It is possible for a function to cross through a horizontal asymptote in between the ends!!





2) Finding the Equation of Oblique Asymptotes

We need to use long division to determine the equation of an oblique asymptote, so let's review that quickly.

ex/ Divide ($3x^2$ + 2x - 4) by (x - 2).

Now let's consider the function $f(x) = \frac{3x^2 + 2x - 4}{x - 2}$.

a) What is the equation of the vertical asymptote?

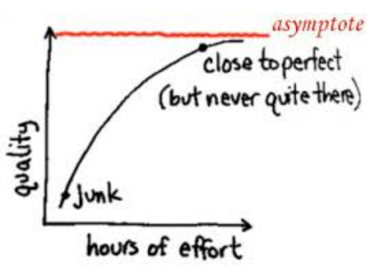
b) Does this have a horizontal asymptote? Why or why not? Use limits to support your answer.

c) Rewrite f(x) in the form f(x) = quotient + $\frac{\text{remainder}}{\text{divisor}}$. Think about the limit as the second portion of this expression approaches infinity. What can you conclude? Will this always be true?

3) Examining the Behaviour of Functions Near Asymptotes

a) How can you check the behaviour of a function as it approaches a vertical asymptote?

b) How can you check the behaviour of a function as it approaches a horizontal asymptote?



4) Curve Sketching (still a work in progress)

- Identify any discontinuities in the domain of your function. Determine the direction from which the curve approaches this asymptote.
- Find the x and y intercepts for the function.
- Take the derivative to find critical values, and plug these in to find critical points.
- Use the first derivative test to classify critical points.
- Test the end behaviour of the function (especially for rational functions) by determining limits at positive and negative infinity. (Is there a horizontal asymptote? an oblique asymptote? Where are they?)
- Identify all intervals of increase/decrease (use an interval table, or some other organized approach to analyzing tangent slope).
- Sketch the curve.

Let's try it! Sketch the graph of $f(x) = \dfrac{2x^2 + 5x + 2}{x + 3}$. Include all of the steps for curve sketching!!