Date: ____________________

4.1 Increasing and Decreasing Functions

4.2 Critical Points, Local Maxima, and Local Minima

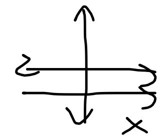
In this unit we will put together everything that we have learned so far, and make some connections to the material learned in Advanced Functions.

Quick Review of Advanced Functions:

What do we mean when we say that a function is increasing on an interval? decreasing on an interval?

↑ x and y increase together

↓ x increases but y decreases



How can we write an interval of increase or decrease?

$x \in (\ , \ )$ $x \in [\ , \ )$ Set notation. cp

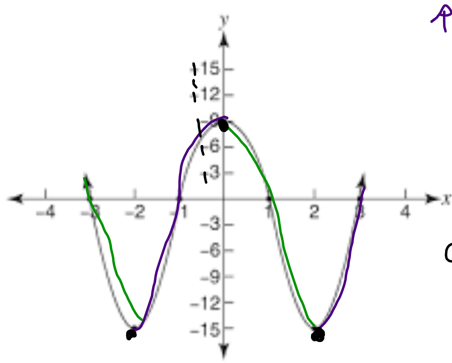
$x \in [\ , \ ]$ Interval notation

What are "critical points"? What are local maxima or local minima? cp

#'s in the domain where $f'(x) = 0$ or is undefined

Values on an interval where a function changes from ↑ to ↓ or vice versa

ex/ For the graphs shown below, state the intervals of increase and the intervals of decrease, as well as the approximate coordinates of all critical points.

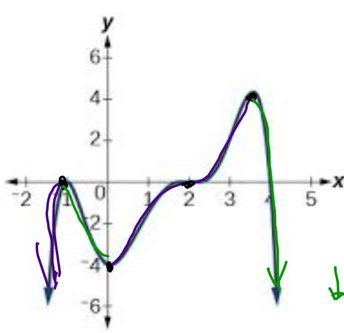


↑ $x \in (-2, 0), (2, \infty)$

↓ $x \in (-\infty, -2), (0, 2)$

CPs: $(-2, -15)$, $(2, -15)$
(min) (min)

$(0, 9)$ max



↑ $x \in (-\infty, -1), (0, 2)$
$(2, 3.5)$

↓ $x \in (-1, 0), (3.5, \infty)$

CP: $(-1, 0)$, $(0, -4)$, $(3.5, 4)$
$(2, 0)$ ← Point of Inflection

Making Connections to Calculus

How can we use what we know to identify our critical points?

Take the derivative of the function & set it equal to zero.

How can we use the derivative of a function to determine if that point is a local maximum or minimum?

Look at a value to each side of the CP & use the signs to determine behaviour.

ex/ Determine the critical points for the function $f(x) = 2x^3 - 3x^2 - 72x$, and use the tangent slope to determine the intervals of increase and decrease. What additional information might be helpful to accurately sketch the graph?

$f'(x) = 6x^2 - 6x - 72$

$6(x^2 - x - 12) = 0$

$6(x-4)(x+3) = 0$

C.Ps: $x = 4$ ← min, $x = -3$ ← max

$f(4) = -208$ $f(-3) = 135$

$(4, -208)$ $(-3, 135)$

For $x = 4$

$f'(3) = -36$ $f'(5) = 48$

Minimum

For $x = -3$

$f'(-4) = 48$ $f'(-2) = -36$

Max

$\uparrow x \in (-\infty, -3), (4, \infty)$

$\downarrow x \in (-3, 4)$

- zeros, y-int

Is it possible for our critical values to represent neither a maximum or a minimum? Explain.

Point of inflection → continue to ↑ or ↓ after this value

ex/ Determine the value of x where the tangent is horizontal for the function $g(x) = x^3 - 3x^2 + 3x + 1$.

$g'(x) = 3x^2 - 6x + 3$

$= 3(x^2 - 2x + 1)$

$0 = 3(x-1)^2$

$x = 1$ ← tangent is horizontal

$\left.\begin{array}{l} g'(0) = 3 \\ g'(2) = 3 \end{array}\right\}$ both (+), so ↑.

Sometimes we can identify critical numbers that DO NOT produce points on our function. What do you think that these values tell us?

Tangent slope is not defined

· cusp · discontinuity

ex/ For the function $f(x) = (x+3)^{\frac{2}{3}}$ determine the critical numbers. Examine the tangent slope for values on each side of the critical number and predict what the point could be.

$$f'(x) = \frac{2}{3}(x+3)^{-1/3}$$
$$= \frac{2}{3(x+3)^{1/3}}$$

$x \neq -3 \leftarrow$ critical number

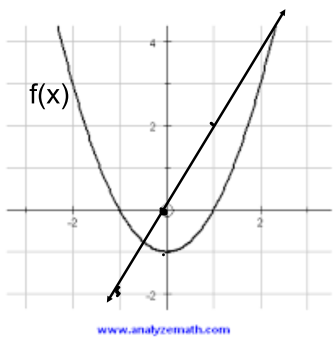
$f(-3) = 0$

$f'(-4) = (-)$ ↓

$f'(-2) = (+)$ ↑

Graphing Functions and their Derivatives

1) Reasoning About Graphs of Functions and Their Derivatives

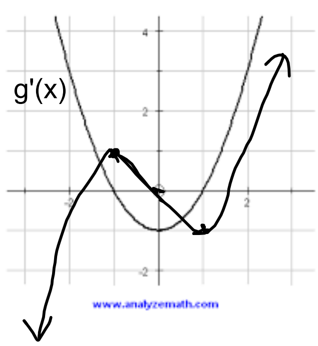


What do you expect the graph of f'(x) to look like? What characteristics of the graph would you use to help you graph the derivative?

linear, assuming f(x) is a parabola

Turning point → zero

$f(x) = x^2 - 1$ $\quad f'(x) = 2x$



What would you expect the graph of g(x) to look like? How can you use the graph of the derivative to graph the original function?

g(x) → cubic.

zeros → turning points

+/- intervals on g'(x) correspond to int of ↑ + ↓ on g(x)

2) Using Derivatives to Graph Accurately

This is called "curve sketching" and will quickly become very important as you continue on in math and science. When we were finding local max and min values, we were conducting the **First Derivative Test** to identify critical numbers.

Steps for Curve Sketching (so far)

- Identify any discontinuities in the domain of your function (we will talk more about asymptotes soon, but you do have some existing knowledge of them!)
- Find the x and y intercepts for the function (Desmos is your friend here sometimes).
- Take the derivative to find critical values.
- Use the first derivative test to classify critical points as maxima, minima, or cusps.
- Sketch the curve.

*Use previous knowledge when you are working with polynomial functions - you know lots of stuff about end behaviours, etc. We have just added in the ability to find the real coordinates of turning points!

One Last Example

Determine values of a, b, and c such that the graph of $y = ax^2 + bx + c$ has a relative minimum at (1, -9) and a y - intercept of (0, -5).

c $\quad c = -5$

$-9 = a(1)^2 + b(1) - 5$

$y = ax^2 + bx - 5$

$-4 = a + b$ (1)

Sub (2) into (1)

$y' = 2ax + b$

$-4 = a - 2a$

When $x = 1$, $y' = 0$

$-4 = -a$

$2a + b = 0$

$b = -2a$ (2)

$4 = a$

$b = -8$

$$\boxed{\begin{aligned} \therefore a &= 4 \\ b &= -8 \\ c &= -5 \end{aligned}}$$