Date: ____________________

3.4 Optimization Problems in Economics and Science

The process for solving optimization problems applied to economics and science is the same as the one that we applied with problems that applied to measurement in 3.3. As long as you know the subject specific vocabulary, you will be able to set up and solve a system of equations to find optimal values.

A Familiar Example:

A bus company has 4000 passengers daily, each paying a fare of $2. For each $0.15 increase in fare, the company estimates that it will lose 40 passengers per day. The company needs to make at least $10450 per day. What fare should be charged to maximize their revenue? R = price x quantity

Use Previous Knowledge:

Let x be the # of $0.15 fare increases.

$R(x) = (2 + 0.15x)(4000 - 40x)$

Zeros: $0.15x = -2$

$x = -13\frac{1}{3}$

Vertex: $x = \frac{100 - 13\frac{1}{3}}{2}$

$= 43\frac{1}{3}$

max

Fare:

$2 + 0.15(43\frac{1}{3})$

$= \$8.50$

$40x = 4000$

$x = 100$

$R(43\frac{1}{3}) = 19266.67$

Use Calculus: $R(x) = (2 + 0.15x)(4000 - 40x)$

$R'(x) = 0.15(4000 - 40x) - 40(2 + 0.15x)$

$= 600 - 6x - 80 - 6x$

$0 = -12x + 520$

$12x = 520$

$x = 43\frac{1}{3}$

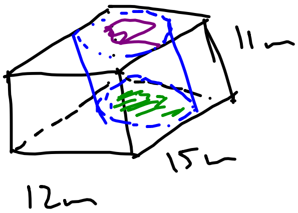
Remember that you need to find the domain of the function that you are taking the derivative of so that you can check the endpoints to make sure that you found the max/min (for unfamiliar functions especially!).

A Few More Examples

1) Minimizing Cost

A cylindrical chemical storage tank with a capacity of 1000 m^3 is going to be constructed in a warehouse that is 12 m by 15 m, with a height of 11 m. The specifications call for the base to be made from sheet steel that costs \$100/m^2 , the top to be made from sheet steel that costs \$50/m^2, and the wall to be made from sheet steel that costs \$80/m^2.

a) Determine whether or not it is possible to construct a tank that will fit inside the warehouse, and use this to determine the constraints on the domain of your cost function.



$V = \pi r^2 h$

① Check $r = 6$

$36\pi h = 1000$

$h = 8.84$ m

② Check $h = 11$

$11\pi r^2 = 1000$

$r^2 = \sqrt{\frac{1000}{11\pi}}$

$r = 5.38$ m

$0 \le r \le 6$

$0 \le h \le 11$

b) Determine the proportions that minimize the cost of the steel for construction.

$\boxed{5.38 \le r \le 6}$

① $C = 100\pi r^2 + 50\pi r^2 + 80(2\pi r h)$

② $\pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

$C(r) = 150\pi r^2 + 160\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

$= 150\pi r^2 + \frac{160\,000}{r}$

min. Cost

$C'(r) = 300\pi r - \frac{160\,000}{r^2}$

Solve

$0 = 300\pi r^3 - 160\,000$

$160\,000 = 300\pi r^3$

$\sqrt[3]{\frac{160\,000}{300\pi}} = r$

$5.54\text{ m} = r$

$C(5.54) = 43344$

$C(5.38) = 43380$

$C(6) = 43631$

2) Maximizing Profit

Through market research, a computer manufacturer found that x thousand units of its new laptop will sell at a price of 2000 - 5x dollars per unit. The cost, C, in dollars to produce this many units is $C(x) = 15\ 000\ 000 + 1\ 800\ 000x + 75x^2$. Determine the level of sales that will maximize profits.

$R(x) = (2000 - 5x)(1000x)$

$P(x) = R(x) - C(x)$

$= 2\ 000\ 000x - 5000x^2 - 15\ 000\ 000 - 1\ 800\ 000x - 75x^2$

$= -5075x^2 + 200000x - 15000000$

$P'(x) = -10150x + 200000$

$10150x = 200\ 000$

$x = 19.704$

∴ To maximize profit, they need to sell 19 704 units.

Business Vocabulary

Recall that **revenue** is **price times quantity**, and **profit** is **revenue minus cost**. **Cost** can be calculated by multiplying the cost per item with the number of items sold. **Marginal Cost** is the rate of change of cost per item sold (C'(x)).