

Date: \_\_\_\_\_



### 3.1 Higher Order Derivatives, Velocity, and Acceleration

As we have already discussed informally, functions have higher order derivatives. We have been taking the first derivative of functions thus far, but we can also take the second derivative by taking the derivative of the first derivative, and so on.

Notation for Second Derivatives:

$$f''(x) \quad \frac{d^2y}{dx^2} \quad \frac{d^2}{dx^2}[f(x)], \quad y''$$

Determine the second derivative of  $f(x) = \frac{x}{3-x}$ . There is not a shortcut - **YOU MUST FIND THE FIRST DERIVATIVE TO FIND THE SECOND!!**

$$\begin{aligned} \textcircled{1} f'(x) &= \frac{(1)(3-x) - x(-1)}{(3-x)^2} & \textcircled{2} f''(x) &= \frac{(0)(3-x)^2 - 3(2(3-x)(-1))}{[(3-x)^2]^2} \\ &= \frac{3-x+x}{(3-x)^2} & &= \frac{6(3-x)}{(3-x)^4} \\ &= \frac{3}{(3-x)^2} & &= \frac{6}{(3-x)^3} \end{aligned}$$



## Connecting Higher Order Derivatives to Straight Line Motion

When an object is moving along a straight line, we can use its position function,  $s(t)$ , to determine its velocity,  $s'(t)$ , and acceleration,  $s''(t)$ .

$$v(t) = s'(t) \quad a(t) = v'(t) \text{ or } a(t) = s''(t)$$

What would this look like in Leibniz notation?

$$v(t) = \frac{ds}{dt} \quad a(t) = \frac{dv}{dt} \quad \text{or } a(t) = \frac{d^2s}{dt^2}$$

### A Brief Moment of Physics:

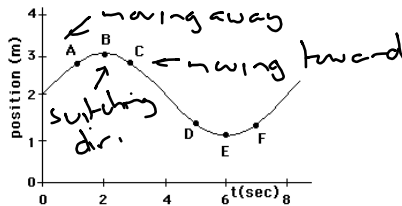
What is the difference between speed and velocity?

Velocity has direction

What is acceleration?

Rate of change of velocity

Also, keep in mind that  $s(t)$  represents the distance of an object from its starting point (origin). When  $s(t)$  is increasing the object is moving away from the origin, and when  $s(t)$  is decreasing, the object is moving toward the origin.



The position of an object moving along a straight line is given by  $s(t) = 6t^2 - t^3$ ,  $t \geq 0$ , where  $s$  is meters and  $t$  is seconds.

- a) Determine the velocity and acceleration of the object at  $t = 2$ . Interpret your solutions.

$$v(t) = 12t - 3t^2 \quad a(t) = 12 - 6t$$

$$v(2) = 12(2) - 3(2)^2 \quad a(2) = 12 - 6(2) \\ = 12 \text{ m/s} \quad = 0 \text{ m/s}^2$$

$\therefore$  at  $t = 2$ , velocity is constant at  $12 \text{ m/s}$  away from the origin.

- b) At what times is the object at rest?

$$v(t) = 0 \quad -3t^2 + 12t = 0$$

$$-3t(t - 4) = 0$$

$$t = 0, t = 4 \text{ s}$$

- c) When does the object return to its initial position?

$$s(t) = 0$$

$$6t^2 - t^3 = 0$$

$$t^2(6 - t) = 0$$

$$t = 0 \quad t = 6$$

$\leftarrow$  It returns after 6s.

