Thursday, February 20, 2020

2.2 The Derivatives of Polynomial Functions

Bellwork:

Find the derivative of each of the following functions from first principles. What do you notice?

1) $f(x) = 2x^2 + x$

2) $g(x) = -x^3 + x$

Examine the following functions and their derivatives. Does it support what you noticed?

$f(x) = 4x^3 - 3x^2 + 5$; $f'(x) = 12x^2 - 6x$

$g(x) = 2x^4 + x^3 + 5x^2 - 2x$; $g'(x) = 8x^3 + 3x^2 + 10x - 2$

$h(x) = 4x^{-2} + x^{-1} + 3x^2$; $h'(x) = -8x^{-3} - x^{-2} + 6x$

We do not always (or ever really...) need to calculate derivatives from first principles. Today we will start looking at derivative rules that apply to polynomial functions. A power function is a function that can be written in the form $f(x) = x^n$. Keep in mind that you can rewrite radical and rational expressions as powers!

Example: Write $\sqrt{x^3}$ and $1/x^4$ as power functions.

Derivative Rules

1) The **constant function rule** states that the derivative of a constant is zero.

$f(x) = c, f'(x) = 0$ $\qquad$ $f(x) = 6, f'(x) = 0$

2) The **power rule** states that for a power function, $f(x) = x^n$, $f'(x) = nx^{n-1}$. Basically, we bring the exponent in front and then subtract one.

3) The **constant multiple rule** states that if $f(x) = kg(x)$ where k is a constant, then $f'(x) = kg'(x)$.

Proof of the Constant Multiple Rule:

Let $f(x) = kg(x)$, and differentiate from first principles.

4) The **sum rule** states that if functions $p(x)$ and $q(x)$ are differentiable, and $f(x) = p(x) + q(x)$, then $f'(x) = p'(x) + q'(x)$.

5) The **difference rule** states that if functions $p(x)$ and $q(x)$ are differentiable, and $f(x) = p(x) - q(x)$, then $f'(x) = p'(x) - q'(x)$.

Proof of the Difference Rule:

Let $f(x) = p(x) - q(x)$ and differentiate from first principles.

Practice Problems:

1) Differentiate the following functions.

a) $f(x) = 5x^2 - 9x^3$

b) $y = \frac{1}{x^3} - \frac{5}{x} + x^{-2}$

c) $h(x) = \sqrt{2x} + \sqrt[3]{x} - 3x^{\frac{2}{5}}$

2) Determine the equation of the tangent to $y = -x^2 + 3x - 5$ at $x = -3$.

3) Determine the point(s) on the graph $f(x) = -x^3 + 3x^2 - 2$ where the tangent(s) are horizontal. (Think about what we know about polynomial functions!)