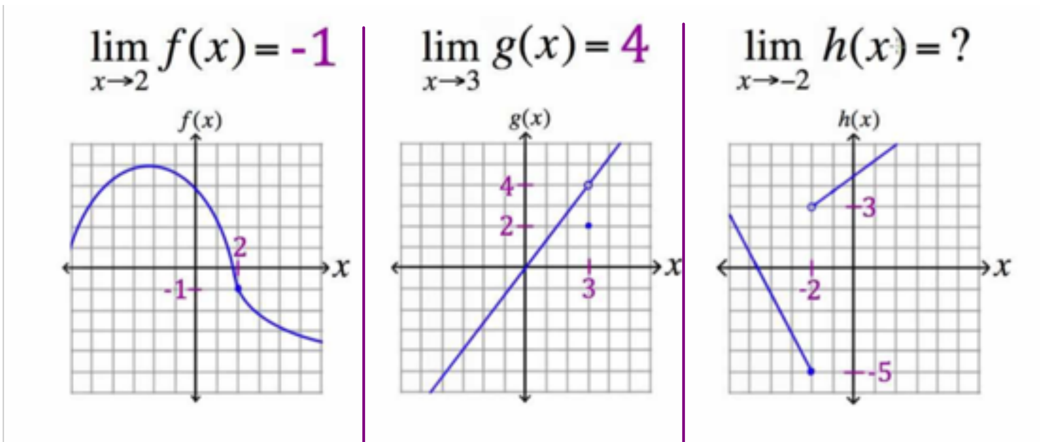
Thursday, February 6, 2020

1.4 The Limit of a Function

Sometimes we are looking for the limiting value for a function rather than the tangent slope to find rate of change. We state this using the notation:

$$\lim_{x \to a} f(x) = L$$

This states that the limit of f(x) as x approaches a equals L. This limit only exists if the value of L is the same when you approach a from the right and the left. This is much easier to see on a graph:



To evaluate limits of functions (remember that we are getting very close to 'a', but not reaching 'a'), we can create a table of values using values that approach 'a', we can graph the function and look at it, or we can use substitution (plug 'a' in to the equation).

Notation Note:

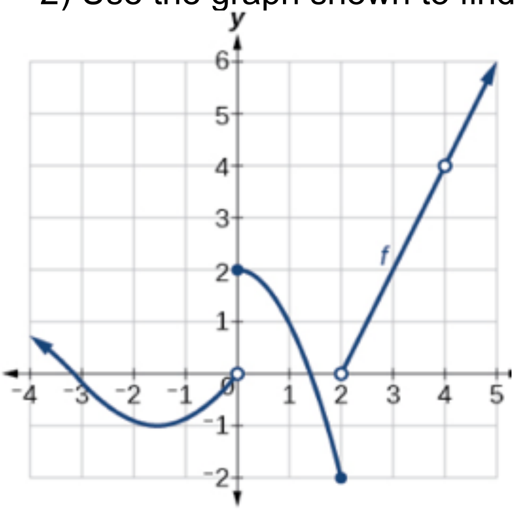
- When we approach a limiting value from the positive side (x is decreasing) we use: $$\lim_{x \to a^+} f(x) = L$$
- When we approach a limiting value from the negative side (x is increasing), we use: $$\lim_{x \to a^-} f(x) = L$$

Practice Problems

1) Calculate the limit for each function.

a. $\lim_{x \to 4} x$ b. $\lim_{x \to -1} (2 - 5x^2)$ c. $\lim_{x \to 3} 3^x$

2) Use the graph shown to find the limits, if they exist.



a. $\lim_{x \to 0^-} f(x)$ b. $\lim_{x \to 0^+} f(x)$ c. $\lim_{x \to 0} f(x)$

g. $\lim_{x \to 4^-} f(x)$ h. $\lim_{x \to 4^+} f(x)$ i. $\lim_{x \to 4} f(x)$

3) Sketch the graph of the piecewise function given below, and then determine the indicated limit, if it exists.

$$f(x) = \begin{cases} x, if\ x \leq -2 \\ x^2 - 3, if\ -2 < x \leq 1 \\ -2x + 4, if\ x > 1 \end{cases} ; \lim_{x \to -2} f(x)$$