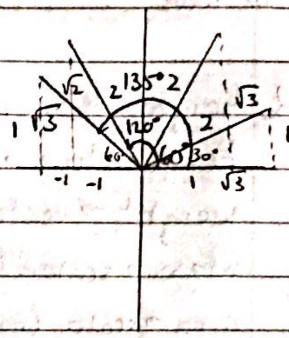
Chapter 6: Introduction to Vectors

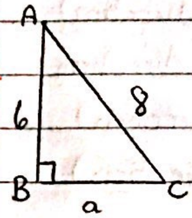
Review of Prerequisite Skills

1a) $\sin 60° = \frac{\sqrt{3}}{2}$ c) $\cos 60° = \frac{1}{2}$ e) $\sin 135° = \frac{\sqrt{2}}{2}$

b) $\tan 120° = -\sqrt{3}$ d) $\cos 30° = \frac{\sqrt{3}}{2}$ f) $\tan 45° = 1$



2.



$a^2 = 8^2 - 6^2$

$a^2 = 28$

$a = \sqrt{28}$ or $2\sqrt{7}$

$\tan A = \frac{2\sqrt{7}}{6}$

$= \sqrt{7}/3$

3a) $c^2 = 37^2 - 22^2$

$c = 29.7$

$\sin B = 22/37$

$B = 36.5°$

$\cos C = 22/37$

$C = 53.5°$

b) $s^2 = 8^2 + 10^2 - 2(8)(10)\cos B$

$\frac{139}{160} = \cos B$

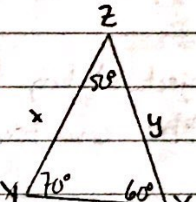
$B = 29.7°$

$\frac{\sin A}{10} = \frac{\sin 29.7°}{5}$

$A = 82.3°$

$C = 68°$

4.



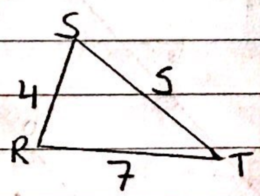
$\frac{x}{\sin 60°} = \frac{6}{\sin 50}$

$x = 6.78$

$\frac{y}{\sin 70°} = \frac{6}{\sin 50°}$

$y = 7.36$

5.



$4^2 = 5^2 + 7^2 - 2(5)(7)\cos T$

$\cos T = 58/70$

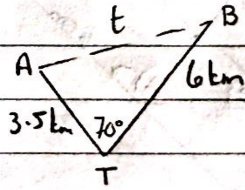
$T = 34°$

$\frac{\sin R}{5} = \frac{\sin 34°}{4}$

$R = 44°$

$S = 102°$

6.

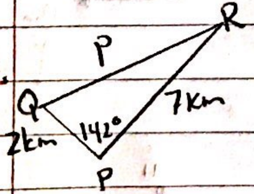


$t^2 = 6^2 + 3.5^2 - 2(6)(3.5)\cos 70°$

$t^2 = 48.25 - 42\cos 70°$

$t = 5.82$ km

7.

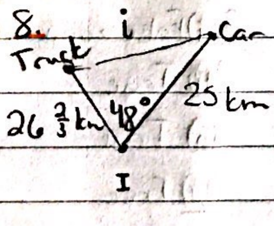


$p^2 = 2^2 + 7^2 - 2(2)(7)\cos 142°$

$p^2 = 53 - 28\cos 142°$

$p = 8.66$ km

8.



100 km/h × 0.25 h = 25 km

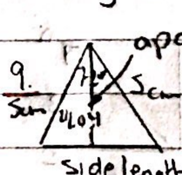
80 km/h × $\frac{1}{3}$ hr = $26\frac{2}{3}$ km

$i^2 = (\frac{80}{3})^2 + (25)^2 - 2(\frac{80}{3})(25)\cos 48$

$i = 21.07$ km

∴ They are 21.1 km apart.

9.



Side = 5.88 cm

$A = \frac{5.88 \times 4.04}{2}$

$= 11.89$ cm²

$A_{prnt} = 59.5$ cm²

6.1 An Introduction to Vectors

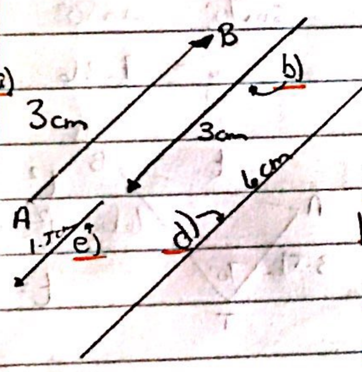
1a) False → could have different directions
b) True
c) False → could have different magnitudes
d) False → the direction has to also be the same or opposite

2. height → scalar (no direction)
temperature → scalar (no direction)
weight → vector (direction)
mass → scalar (no direction)
area → scalar (no direction)
volume → scalar (no direction)
distance → scalar (no direction)
displacement → vector (direction)
speed → scalar (no direction)
force → vector (has direction)
velocity → vector (direction)

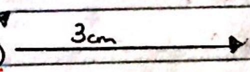
3. An object sliding along a counter stops because friction between the object (→) and the counter (←) slows the initial force of pushing.

4a) BC and AD, AB and DC, AE and EC, DE and EB
b) AB and CD, BC and DA, BE and DE, CE and AE
c) AE and EB, DE and EC
(lots of other correct answers)

5a)

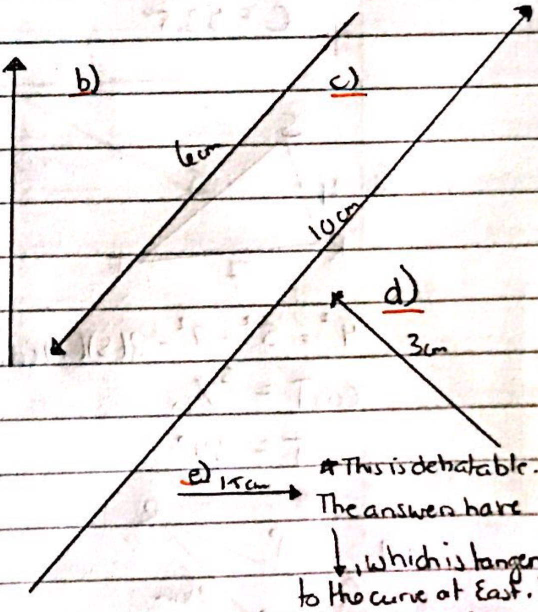


c)



8a) Due south at 400 km/h
b) Southwest at 70 km/h
c) Southeast at 30 km/h
d) Due east at 25 km/h

6a) b) c)

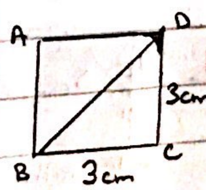


*This is debatable. The answers have ↓, which is tangent to the curve at East. I interpret heading east to be at the bottom of the circle in 6.

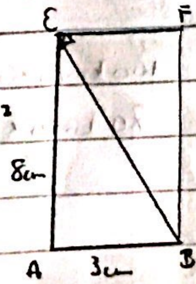
7a) Due south at 100 km/h
b) Due west at 50 km/h
c) Northeast at 100 km/h
d) Northwest at 25 km/h
e) Due east at 60 km/h

9a) i) False → opposites
ii) True
iii) True
iv) True

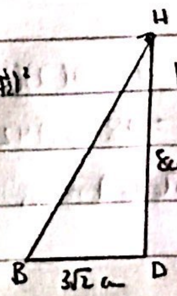
9b)



$|\vec{BD}|^2 = |\vec{BC}|^2 + |\vec{CD}|^2$
$= 3^2 + 3^2$
$|\vec{BD}| = \sqrt{18}$
$= 3\sqrt{2}$



$|\vec{BE}|^2 = |\vec{AB}|^2 + |\vec{AE}|^2$
$= 3^2 + 8^2$
$|\vec{BE}| = \sqrt{73}$



$|\vec{BH}|^2 = |\vec{BD}|^2 + |\vec{DH}|^2$
$= (3\sqrt{2})^2 + 8^2$
$|\vec{BH}| = \sqrt{82}$

Scanned with CamScanner

10a) There are multiple forces working here. The velocity at a given instant is in a straight line (tangent to the curve).

b) Speed

c) The direction changes at every point, even if the rate he runs at is constant.

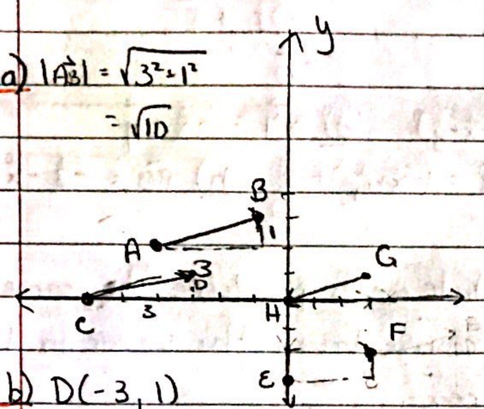
d) C.

e) 7/8 of the circle (Occurs between D and A) $t = \frac{d}{s}$, so $t = \frac{0.875}{15}$ $= 0.058\bar{3}$h, or 3.5 min

f) Southwest

11a) $|\vec{AB}| = \sqrt{3^2+1^2}$

$= \sqrt{10}$

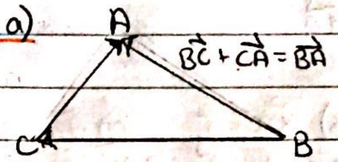


b) D(-3, 1)

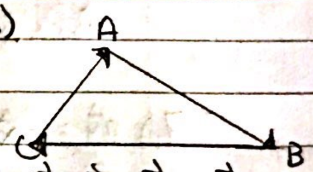
c) E(0, -3)

d) H(0, 0)

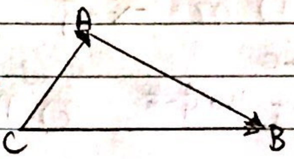
2. a)



b)



c)

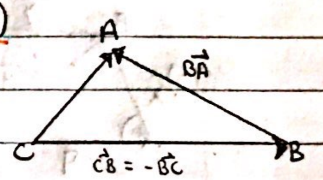


$\vec{AB} - \vec{AC}$

$= \vec{AB} + \vec{CA}$

$= \vec{CB}$

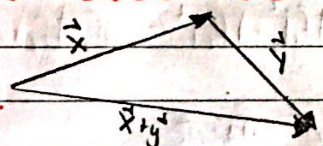
d)



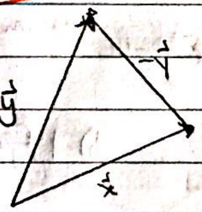
$-\vec{BC} + \vec{BA} = \vec{CA}$

6.2 Vector Addition

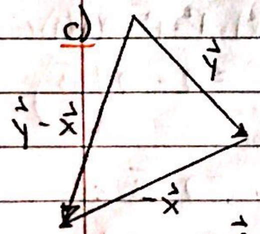
1a)



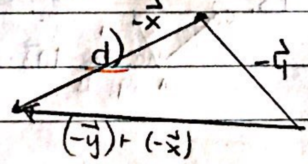
b)



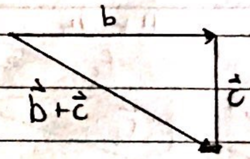
c)

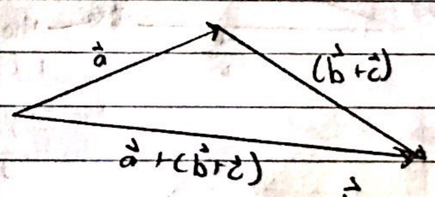


d)

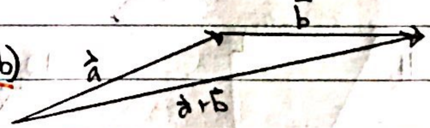


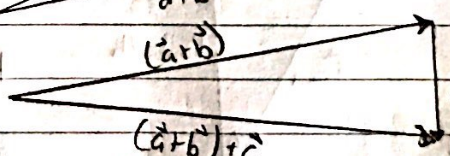
4a)





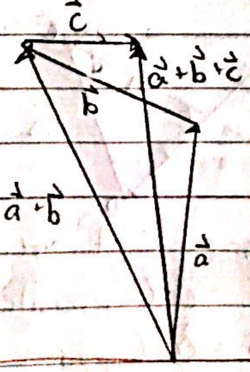
4b)



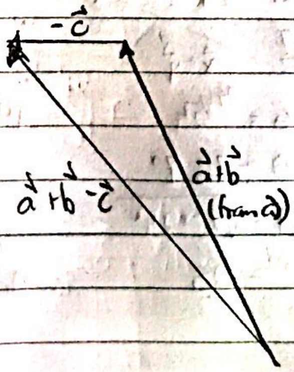


c) $\vec{a} + (\vec{b}+\vec{c}) = (\vec{a}+\vec{b}) + \vec{c}$

3a)

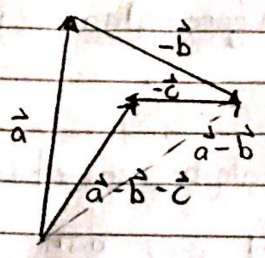


b)

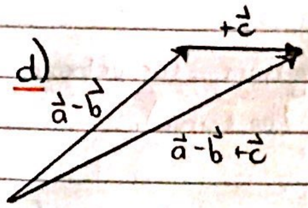


3 is continued on the back. I accidentally did 4 before 3. :)

3c)



d)



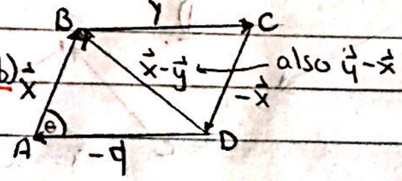
5a) $\vec{PQ} - \vec{RQ} + \vec{RS}$
$= \vec{PQ} + \vec{QR} + \vec{RS}$
$= \vec{PR} + \vec{RS}$
$= \vec{PS}$

b) $\vec{PS} + \vec{RQ} - \vec{RS} - \vec{PQ}$
$= \vec{QP} + \vec{PS} + \vec{SR} + \vec{RQ}$
$= \vec{QS} + \vec{SR} + \vec{RQ}$
$= \vec{QR} + \vec{RQ}$
$= \vec{0}$

6. $\vec{x}+\vec{y} = \vec{MS}$, $\vec{z}+\vec{t} = \vec{SQ}$
$\vec{MS} + \vec{SQ} = \vec{MQ}$

7a) $\vec{BY} = -\vec{x}$ b) $\vec{XB} = \vec{y}$ c) $\vec{OB} = \vec{y} + \vec{x}$ (or $\vec{x}+\vec{y}$) d) $\vec{XY} = -\vec{x} + \vec{y}$
e) $\vec{OQ} = (\vec{x}+\vec{y}) + \vec{z}$ f) $\vec{AZ} = -\vec{x} - \vec{y}$ g) $\vec{XR} = (-\vec{x}+\vec{y}) + \vec{z}$ h) $\vec{PO} = -\vec{z} - \vec{x}$

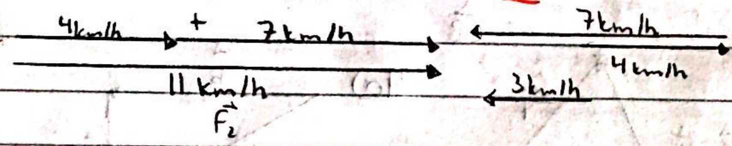
8a) b)



c) The two statements produce the same vector, so the cosine law is the same in both instances.

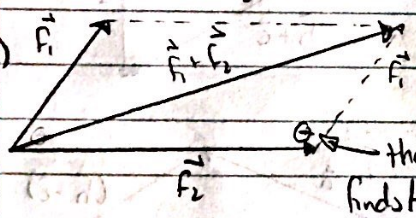
9a) 11 km/h

b)



c) 3km/h

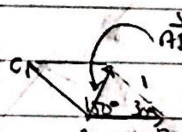
10a)



the cosine law finds this angle, not the angle formed at the tugboat.

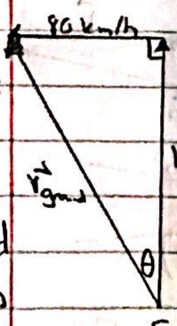
b) $|\vec{F_1}+\vec{F_2}|^2 = |\vec{F_1}|^2 + |\vec{F_2}|^2 - 2|\vec{F_1}||\vec{F_2}|\cos\theta$

13.



$|\vec{AB}+\vec{AC}|^2 = 1^2 + 1^2 - 2(1)(1)\cos 30°$
$= 2 - 2\left(\frac{\sqrt{3}}{2}\right)$
$= 2 - \sqrt{3}$
$c^2 = 0.27$

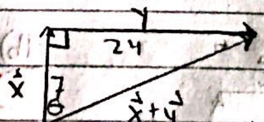
11.



$\therefore$ The ground velocity is 170 km/h, N 28° W.

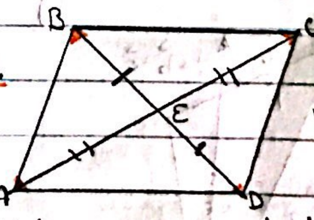
$\tan\theta = 8/15$
$\theta = 28°$
$|\vec{v}| = \sqrt{150^2 + 80^2}$
$= 170$ km/h

12.



$\tan\theta = 24/7$
$\theta = 73.7°$
$|\vec{x}+\vec{y}| = \sqrt{7^2 + 24^2}$
$= 25$

14.



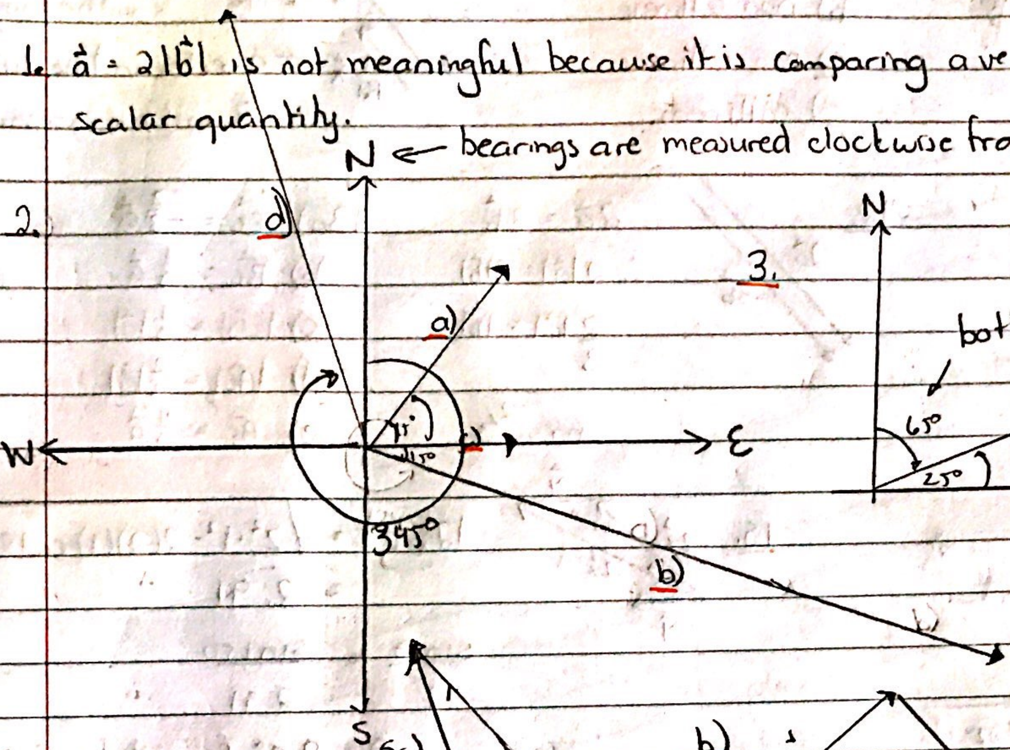
Diagonals bisect each other

Addition is commutative.
$\vec{EA} + \vec{EC} = \vec{0}$ (opposite vectors)
$\vec{EB} + \vec{ED} = \vec{0}$ (opposite vectors)
$\therefore \vec{EA} + \vec{EB} + \vec{EC} + \vec{ED} = \vec{0}$

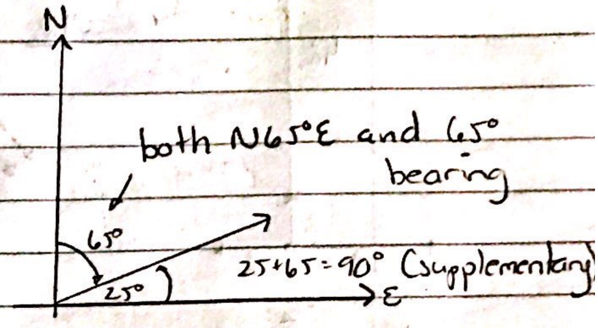
6.3 Multiplication of a Vector by a Scalar

1. $\vec{a} = 2|\vec{b}|$ is not meaningful because it is comparing a vector quantity with a scalar quantity.

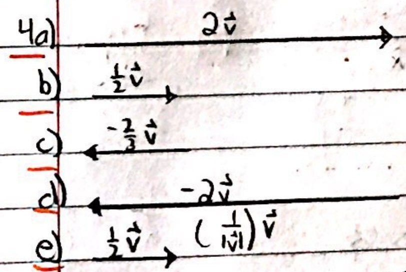
2.



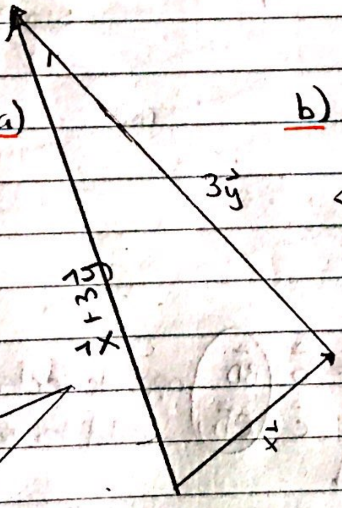
3.



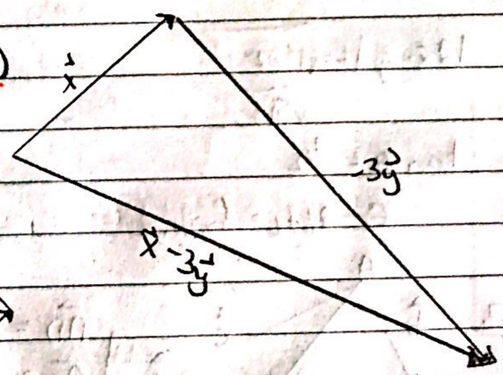
4a)



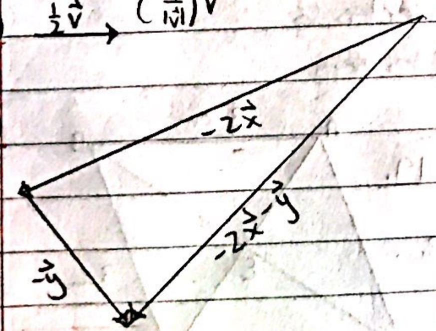
5a)



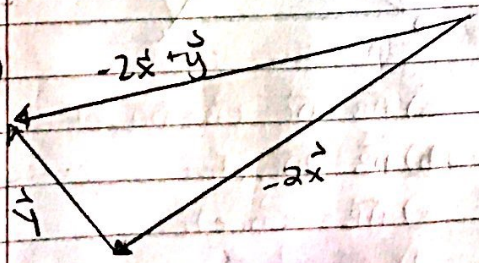
b)



5d)



5c)



6. Answers will vary. Do your best + measure w/ a ruler!

7a) $\frac{1}{2}\vec{c} = \frac{2}{3}\vec{b}$

$3\vec{c} = 4\vec{b}$

$-4\vec{b} + 3\vec{c} = \vec{0}$

$m = -4,\ n = 3$

There are infinite possibilities as long as the ratio is the same.

b) $\vec{a} = \frac{1}{2}\vec{c}$

$2\vec{a} = \vec{c}$

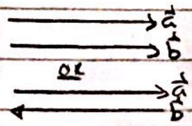
$\therefore 2\vec{a} - \vec{c} = \vec{0}$

$\boxed{\vec{a} = \frac{2}{3}\vec{b} \quad \text{Let } e = 0}$

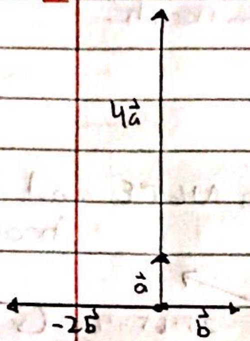
$d\vec{a} + e\vec{b} + \vec{c} = \vec{0}$

When $d = 2$, $e = 0$ and $f = -1$

Again infinite possibilities.

8. 

9. Yes.

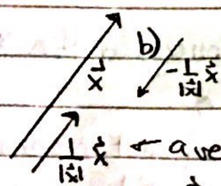


10a) collinear

b) non-collinear

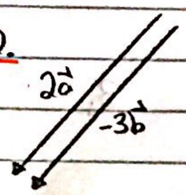
c) non-collinear

d) collinear

11. 

b) $-\frac{1}{|\vec{x}|}\vec{x}$ ← a vector in the opposite direction of $\vec{x}$ with a magnitude of 1

a) $\frac{1}{|\vec{x}|}\vec{x}$ ← a vector in the direction of $\vec{x}$ with a magnitude of 1.

12. 

$2\vec{a} = 3\vec{b}$

$\therefore 2|\vec{a}| = 3|\vec{b}|$

$\frac{2}{3}|\vec{a}| = |\vec{b}|$

$m = 2/3$

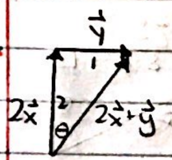
13a) $\vec{EC} = -\frac{2}{3}\vec{a}$

b) $\vec{BC} = \frac{1}{3}\vec{a}$

c) $|\vec{ED}| = \frac{1}{3}|\vec{a}|$

d) $|\vec{AC}| = \frac{2}{3}|\vec{a}|$

e) $\vec{AE} = \frac{4}{3}\vec{a}$

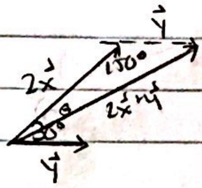
14. 

$|2\vec{x}+\vec{y}| = \sqrt{1^2+2^2}$

$= \sqrt{5}$

$\tan\theta = 1/2$

$\theta = 26.6°$ from $\vec{x}$

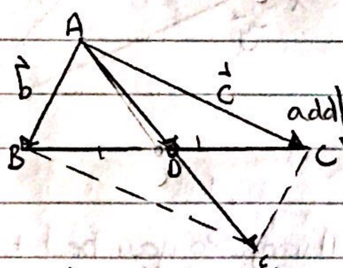
15. 

$|2\vec{x}+\vec{y}| = \sqrt{2^2+1^2-2(2)(1)\cos 150°}$

$= 2.91$

$\frac{\sin\theta}{1} = \frac{\sin 150}{2.91}$

$\theta = 9.9°$ from $\vec{x}$ toward $\vec{y}$

17. 

$\vec{AD} = \vec{b} + \vec{BD}$

add

$\vec{AD} = \vec{c} + \vec{CD}$

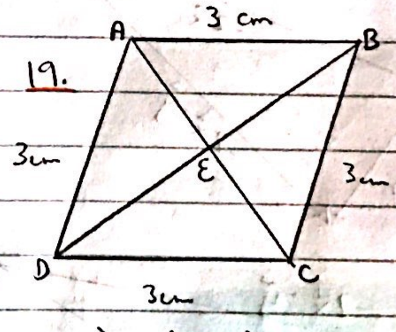
opposite vectors, so add to zero.

$2\vec{AD} = \vec{b} + \vec{c}$

$\vec{AD} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$

$\vec{AE} = \vec{b} + \vec{c}$

$\vec{AE} = 2\vec{AD}$

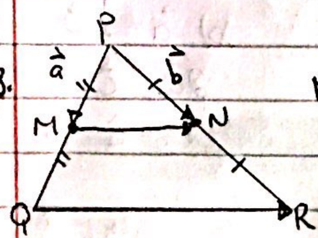
19. 

a) $\vec{AB} = \vec{DC}$, $\vec{CB} = \vec{DA}$, etc.

b) $\vec{DB} = 2\vec{DE}$, etc.

c) $\vec{AB} = -\vec{CD}$, etc.

d) $\vec{AE} = \frac{1}{2}\vec{AC}$, etc.

18. 

$\vec{MN} = -\vec{a} + \vec{b}$

$\vec{QR} = -2\vec{a} + 2\vec{b}$

$\therefore \vec{MN}$ and $\vec{QR}$ are parallel and $|\vec{QR}| = 2|\vec{MN}|$