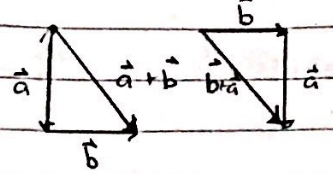
6.4 Properties of Vectors

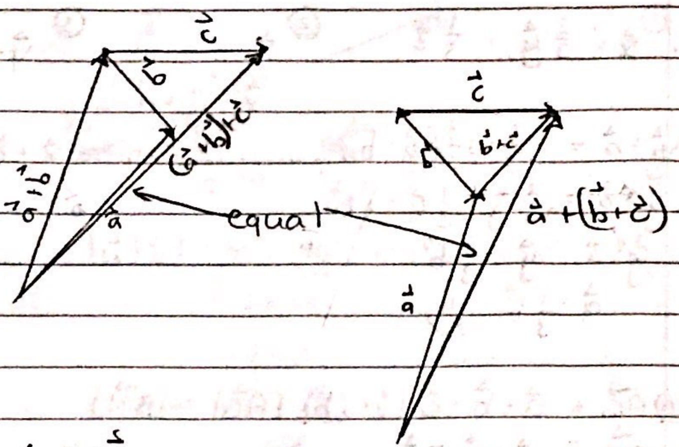
b) 0   b) 1   c) $\vec{0}$ d) 1

2.

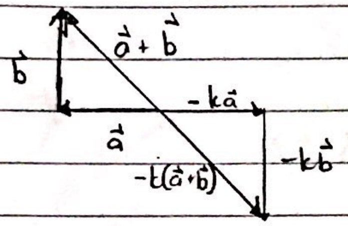


$\vec{a}+\vec{b}=\vec{b}+\vec{a}$

3.



4.



6a) $k\vec{c}$

b) $\vec{0}$

c) Yes, diagonals are equal in length.

7. $3(\vec{a}-2\vec{b}-5\vec{c})-3(2\vec{a}-4\vec{b}+2\vec{c})-(\vec{a}-3\vec{b}+3\vec{c})$

$=3\vec{a}-6\vec{b}-15\vec{c}-6\vec{a}+12\vec{b}-6\vec{c}-\vec{a}+3\vec{b}-3\vec{c}$

$=-4\vec{a}+9\vec{b}-24\vec{c}$

5. $\vec{PQ}=(\vec{RQ}+\vec{SR})+\vec{TS}+\vec{PT}$

$=\vec{SQ}+\vec{TS}+\vec{PT}$

$=\vec{TQ}+\vec{PT}$

$=\vec{PQ}$

$\vec{PQ}=\vec{RQ}+(\vec{SR}+\vec{TS})+\vec{PT}$

$=\vec{RQ}+TR+PT$

$=TQ+\vec{PT}$

$=\vec{PQ}$

$\vec{PQ}=\vec{RQ}+\vec{SR}+(\vec{TS}+\vec{PT})$

$=\vec{RQ}+\vec{SR}+\vec{PS}$

$=\vec{SQ}+\vec{PS}$

$=\vec{PQ}$

8a) $2\vec{a}-3\vec{b}$

$=2(3\vec{i}-4\vec{j}+\vec{k})-3(-2\vec{i}+3\vec{j}-\vec{k})$

$=6\vec{i}-8\vec{j}+2\vec{k}+6\vec{i}-9\vec{j}+3\vec{k}$

$=12\vec{i}-17\vec{j}+5\vec{k}$

b) $\vec{a}+5\vec{b}$

$=3\vec{i}-4\vec{j}+\vec{k}+5(-2\vec{i}+3\vec{j}-\vec{k})$

$=3\vec{i}-4\vec{j}+\vec{k}-10\vec{i}+15\vec{j}-5\vec{k}$

$=-7\vec{i}+11\vec{j}-4\vec{k}$

c) $2(\vec{a}-3\vec{b})-3(-2\vec{a}-7\vec{b})$

$=2\vec{a}-6\vec{b}+6\vec{a}+21\vec{b}$

$=8\vec{a}+15\vec{b}$

$=8(3\vec{i}-4\vec{j}+\vec{k})+15(-2\vec{i}+3\vec{j}-\vec{k})$

$=24\vec{i}-32\vec{j}+8\vec{k}-30\vec{i}+45\vec{j}-15\vec{k}$

$=-6\vec{i}+13\vec{j}-7\vec{k}$

9. $2\vec{x}+3\vec{y}=\vec{a}$ ①, $-\vec{x}-5\vec{y}=6\vec{b}$ ②

$2(-6\vec{b}+5\vec{y})+3\vec{y}=\vec{a}$ $\vec{x}=-6\vec{b}+5\vec{y}$

$-12\vec{b}+10\vec{y}+3\vec{y}=\vec{a}$

$13\vec{y}=12\vec{b}+\vec{a}$ $-\vec{x}+5\left(\frac{1}{13}\vec{a}+\frac{12}{13}\vec{b}\right)=6\vec{b}$

$\boxed{\vec{y}=\frac{1}{13}\vec{a}+\frac{12}{13}\vec{b}}$ $-\vec{x}+\frac{5}{13}\vec{a}+\frac{60}{13}\vec{b}=6\vec{b}$

$\boxed{\vec{x}=\frac{5}{13}\vec{a}-\frac{18}{13}\vec{b}}$

10. ① $\vec{x} = \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z}$ ② $\vec{x} - \vec{y} = \vec{a}$ ③ $\vec{y} - \vec{z} = \vec{b}$ Prove: $\vec{a} = \frac{1}{3}\vec{b}$

$\vec{x} = \vec{y} + \vec{a}$ $\quad$ $\vec{z} = \vec{y} - \vec{b}$

Express $\vec{x}$ + $\vec{z}$ in terms of y

$\vec{y} + \vec{a} = \frac{2}{3}\vec{y} + \frac{1}{3}(\vec{y} - \vec{b})$

$\vec{y} + \vec{a} = \frac{2}{3}\vec{y} + \frac{1}{3}\vec{y} - \frac{1}{3}\vec{b}$

$\cancel{\vec{y}} + \vec{a} = \cancel{\vec{y}} - \frac{1}{3}\vec{b}$

$\vec{a} = \frac{1}{3}\vec{b}$

11 a) $\vec{AG} = \vec{a} + \vec{b} + \vec{c}$ b) $|\vec{AG}| = |\vec{BH}|$

$\vec{BH} = -\vec{a} + \vec{b} + \vec{c}$ Diagonals are

$\vec{CE} = -\vec{a} - \vec{b} + \vec{c}$ equal in length.

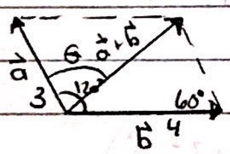
$\vec{DF} = \vec{a} - \vec{b} + \vec{c}$

Mid-Chapter Review

1 a) $\vec{AB} = \vec{DC}$
$\vec{BA} = \vec{CD}$
$\vec{AD} = \vec{BC}$
$\vec{CB} = \vec{DA}$
$\vec{AP} = ?$

b) $|\vec{PD}| = |\vec{BC}|$ because $|\vec{PD}| = |\vec{AD}|$ and $\vec{AD} \parallel \vec{BC}$ with equal magnitude.

2 a) $\vec{RQ} + \vec{RS} = \vec{RV}$
b) $\vec{RQ} + \vec{QV} = \vec{RV}$
c) $\vec{PW} + \vec{WS} = \vec{PS}$
d) $(\vec{RQ} + \vec{RS}) + \vec{VU} = \vec{RU}$
e) $\vec{PW} - \vec{VP} = \vec{PS}$
f) $\vec{PW} + \vec{WR} + \vec{RQ} = \vec{PQ}$

3. 

$|\vec{a} + \vec{b}|^2 = 3^2 + 4^2 - 2(3)(4)(\cos 60°)$

$= \sqrt{25 - 12}$

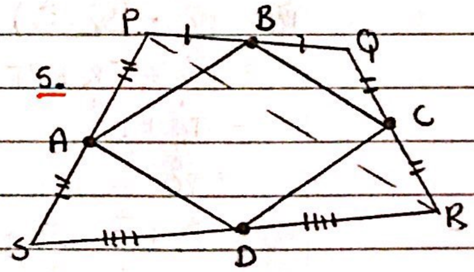
$= \sqrt{13}$

$\frac{\sin\theta}{4} = \frac{\sin 60°}{\sqrt{13}}$

$\theta = 73.9°$

4. $\vec{x} = t\vec{y}$, $|\vec{x}| = 4|\vec{y}|$

$t = \pm 4$

5. 

① $\vec{DA} = \vec{DS} + \vec{SA}$ $\quad |\vec{RP}| = 2|\vec{DA}|$

$\vec{RP} = \vec{RS} + \vec{SP}$ $\quad \vec{RP} \parallel \vec{DA}$

② $\vec{BC} = \vec{BQ} + \vec{CQ}$ $\quad |\vec{PR}| = 2|\vec{BC}|$

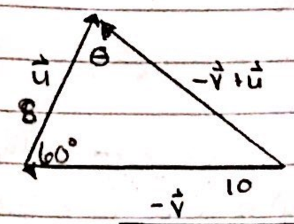
$\vec{PR} = \vec{PQ} + \vec{QR}$ $\quad \vec{PR} \parallel \vec{BC}$

③ ∴ $\vec{AD}$ is parallel to $\vec{BC}$

④ Use the same patterns to show $\vec{AB} \parallel \vec{DC}$

⑤ ∴ ABCD is a parallelogram.

6a)



$|\vec{u}-\vec{v}| = \sqrt{8^2+10^2-2(8)(10)\cos 60^\circ}$

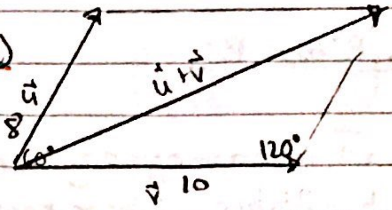
$= \sqrt{84}$

$= 2\sqrt{21}$

b) $\frac{\sin\theta}{10} = \frac{\sin 60^\circ}{2\sqrt{21}}$

$\theta = 70.9^\circ$

c)



$|\vec{u}+\vec{v}| = \sqrt{8^2+10^2-2(8)(10)\cos 120^\circ}$

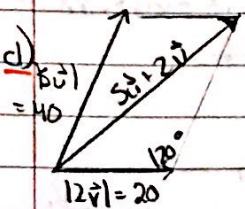
$= \sqrt{244}$

$= 2\sqrt{61}$

$\frac{1}{|\vec{u}+\vec{v}|}(\vec{u}+\vec{v}) = \frac{1}{2\sqrt{61}}(\vec{u}+\vec{v})$

8. $|\vec{m}+\vec{n}| = |\vec{m}|-|\vec{n}|$

d)

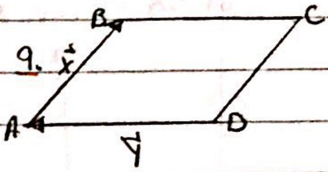


$|5\vec{u}+2\vec{v}| = \sqrt{40^2+20^2-2(40)(20)\cos 120^\circ}$

$= \sqrt{2800}$

$= 20\sqrt{7}$

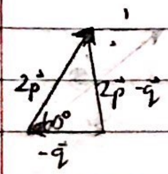
9.



$\vec{BC} = -\vec{y}$ $\vec{DC} = \vec{x}$

$\vec{BD} = -\vec{x}-\vec{y}$ $\vec{AC} = \vec{x}-\vec{y}$

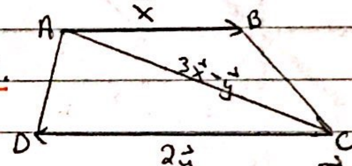
7.



$|2\vec{p}-\vec{q}| = \sqrt{2^2+1^2-2(2)(1)\cos 60^\circ}$

$= \sqrt{3}$

11.



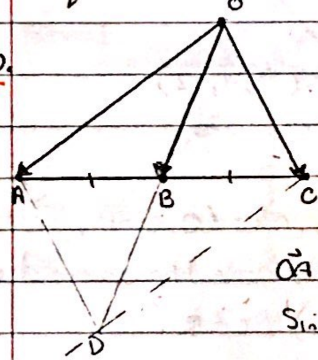
$\vec{BC} = -\vec{x}+3\vec{x}-\vec{y}$

$= 2\vec{x}-\vec{y}$

$\vec{BD} = 2\vec{x}-\vec{y}+2\vec{y}$

$= 2\vec{x}+\vec{y}$

10.



· diagonals of a parallelogram bisect eachother.

12. 460 km/h, south

$\vec{OA}+\vec{AB} = \vec{OB}$, $\vec{OC}+\vec{CB} = \vec{OB}$

Since $\vec{AB} = -\vec{CB}$

$\vec{OA}+\vec{AB} = \vec{OB}$ ①
$\vec{OC}-\vec{AB} = \vec{OB}$ ② } add

$\vec{OA}+\vec{OC} = 2\vec{OB}$ ☺

13a) $\vec{PQ}+\vec{QR}+\vec{RT} = \vec{PT}$

b) $\vec{PQ}+\vec{QR}-\vec{TR}$

$= \vec{PQ}+\vec{QR}+\vec{RT}$

$= \vec{PT}$

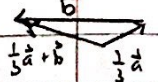
c) $\vec{PR}-(\vec{PT}-\vec{ST})$

$= \vec{PR}-(\vec{PT}+\vec{TS})$

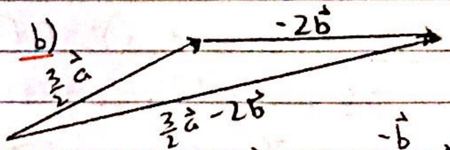
$= \vec{PR}-\vec{PS}$

$= \vec{PR}+\vec{SP}$

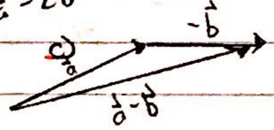
14.a)



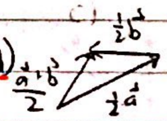
b)



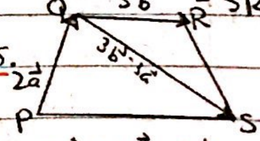
c)



d)



15.



$\vec{PS} = 2\vec{a}+3\vec{b}-3\vec{a}$

$= -\vec{a}+3\vec{b}$

$\vec{RS} = -3\vec{b}+3\vec{b}-3\vec{a}$

$= -3\vec{a}$

6.5 Vectors in R^2 and R^3

1. No because $\sqrt{-1}$ is an imaginary number so it can't be located on the real plane.

2a) Just like in two-space, where each point (x, y) is a unique location, points in 3-space occupy their own unique location given by (x, y, z). The vector from $(0,0,0)$ to a specific point, P, will also be unique.

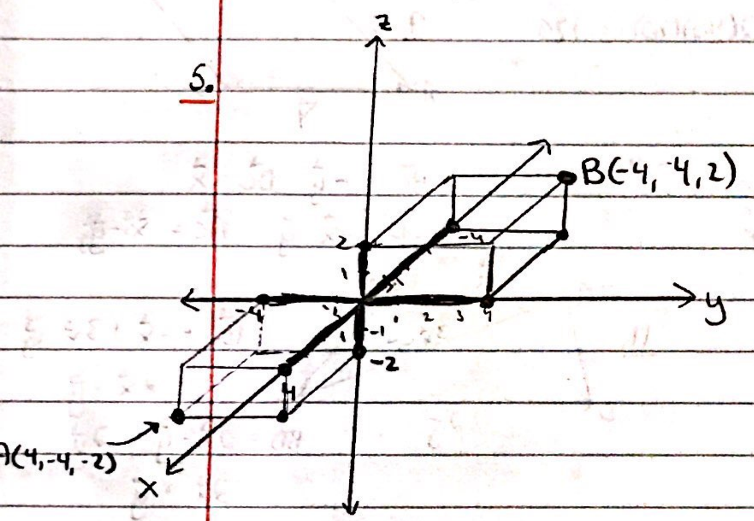
b) $a = -4$, $b = -3$, $c = -8$ because P is a unique point in R^3.

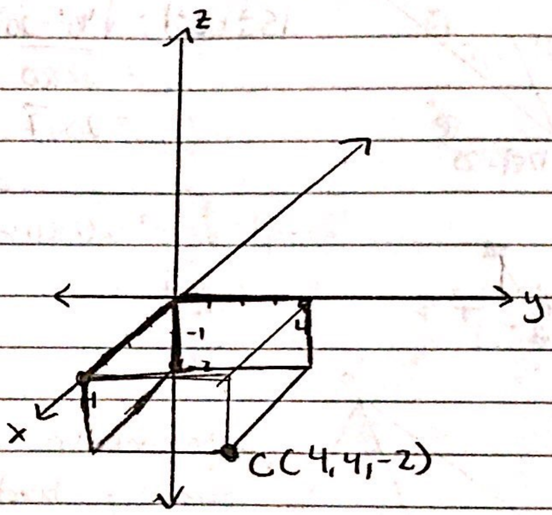
3a) $a = 5$, $b = -3$, $c = 8$

b) $\vec{OA} = (5, -3, 8)$

4. $\sqrt{3}$ is not an integer, so it is not included in I^3, but $\sqrt{3}$ is a real number, so it exists in R^3.

5.





6a) y-axis, $(0, b, 0)$, $b \in R$

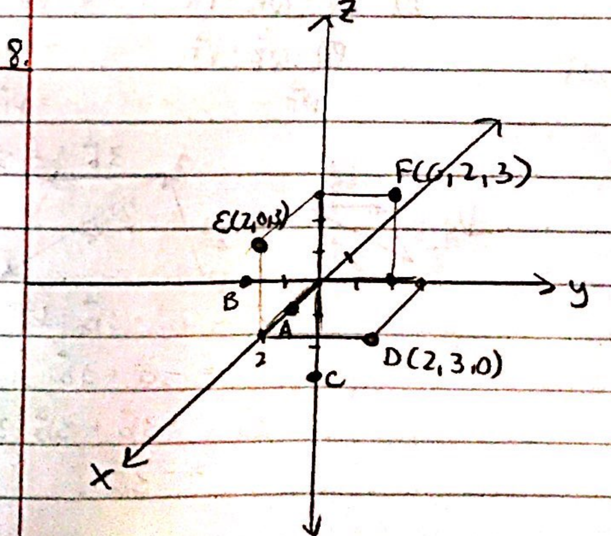
b) $\vec{OA} = (0, -1, 0)$

7a) $\vec{OA} = (0,0,1)$, $\vec{OB} = (0,0,-1)$, $\vec{OC} = (0,0,2)$, etc.

b) Yes, they all have the same direction.

c) $\vec{OP} = (0, 0, c)$, $c \in R$.

8.



★ 9-11 are on the next page!

12. $P(2, a-c, a)$ $Q(2, 6, 11)$

a) $a - c = 6$ $a = 11$

$-c = 6 - 11$

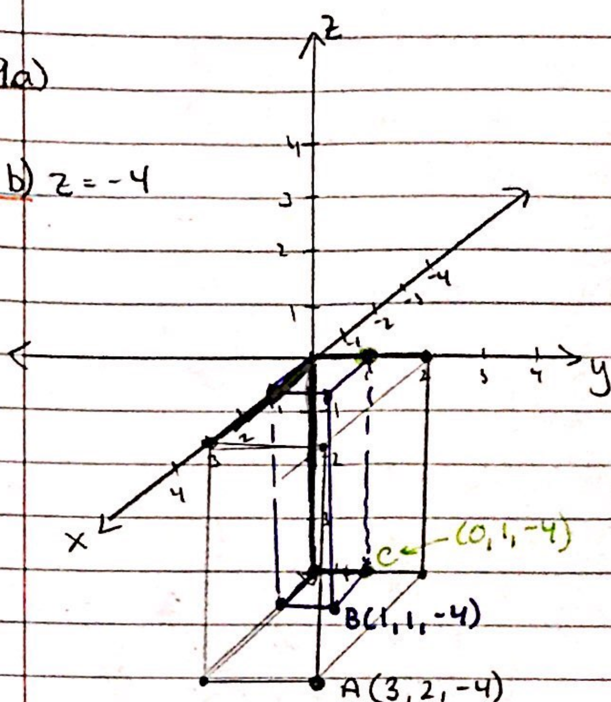
$-c = -5$ $\therefore a = 11, c = 5$

$c = 5$

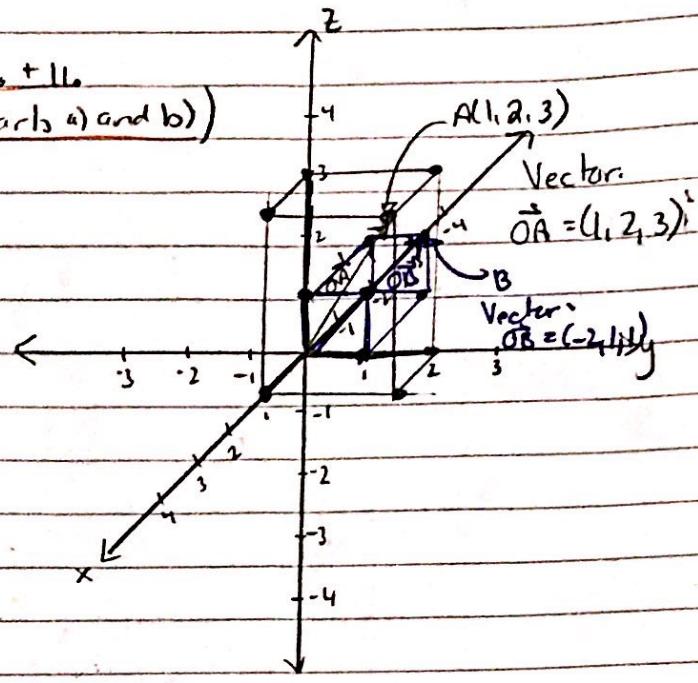
b) $|\vec{OP}| = |\vec{OQ}|$ because they are the same vector.

9a)

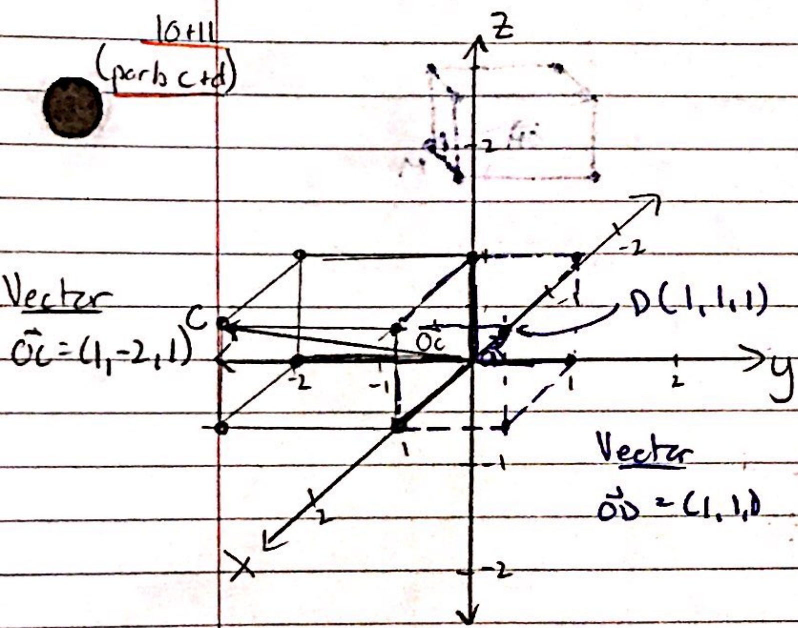
b) $z = -4$



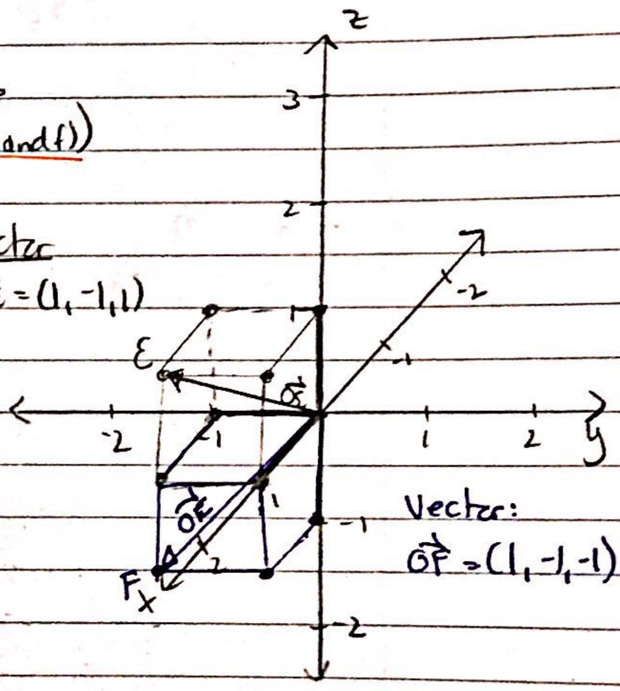
10. + 11.
(Parts a) and b))



10+11
(parts c+d)



10. + 11.
(parts e) and f))



13. $P(x, y, 0)$
xy-plane ($z = 0$)
$Q(x, 0, z)$
xz-plane ($y = 0$)
$R(0, y, z)$
yz-plane ($x = 0$)

14a) $y = 0$ All of the points are on the plane formed by x and z axes (y-coordinate is always zero).

b) The vectors originate at $(0, 0, 0)$, which is on the plane $y = 0$. That means that the vectors lie on the plane.

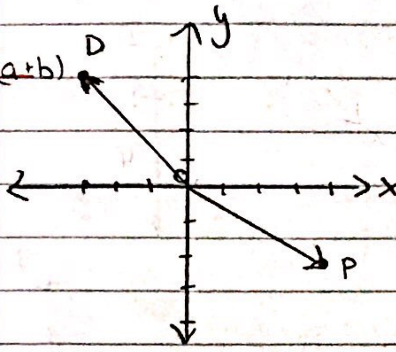
15a) $A(-2,0,0)$ $D(0,0,-7)$ b) $\vec{OA} = (-2,0,0)$ $\vec{OD} = (0,0,-7)$

$B(-2,4,0)$ $E(0,4,-7)$ $\vec{OB} = (-2,4,0)$ $\vec{OE} = (0,4,-7)$

$C(0,4,0)$ $F(-2,0,-7)$ $\vec{OC} = (0,4,0)$ $\vec{OF} = (-2,0,-7)$

c) 7 units d) $y=4$ e) All points with a y-coordinate of 4. $(x, 4, z)$. x has to be between 0 and -2, z has to be between 0 and -7.

16.(a+b)



(c+d)