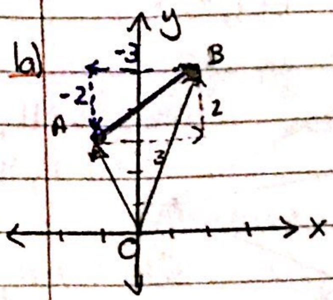
6.6 Operations with Algebraic Vectors in R^2

1a)



$\vec{AB} = (3, 2)$

$\vec{BA} = (-3, -2)$

b) $|\vec{OA}| = \sqrt{(-1)^2 + (3)^2}$

$= \sqrt{10}$

$|\vec{OB}| = \sqrt{2^2 + 5^2}$

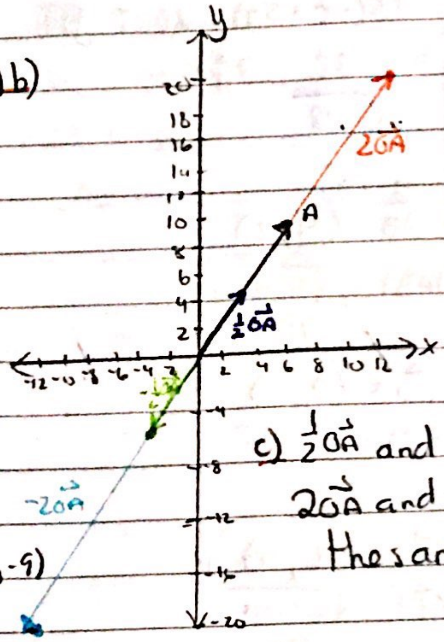
$= \sqrt{29}$

c) $|\vec{AB}| = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$

$|\vec{BA}| = \sqrt{13}$

2a) b)



c) $\frac{1}{2}\vec{OA}$ and $-\frac{1}{2}\vec{OA}$ and $2\vec{OA}$ and $-2\vec{OA}$ have the same magnitudes.

3. $\vec{OA} = 3\vec{i} - 4\vec{j}$

$\vec{OA} = (3, -4)$

$|\vec{OA}| = \sqrt{3^2 + 4^2}$

$= 5$

4a) $a\vec{i} + 5\vec{j} = (-3, b)$

$a = -3, \ b = 5$

b) $|(-3, 5)| = \sqrt{(-3)^2 + 5^2}$

$= \sqrt{34}$

5. $\vec{a} = (-60, 11)$ $\vec{b} = (-40, -9)$

a) $|\vec{a}| = \sqrt{60^2 + 11^2}$

$= 61$

$|\vec{b}| = \sqrt{(-40)^2 + (-9)^2}$

$= 41$

b) $\vec{a} + \vec{b} = (-100, 2)$

$|\vec{a} + \vec{b}| = \sqrt{100^2 + 2^2}$

$= 2\sqrt{2501}$

$\vec{a} - \vec{b} = (-20, 20)$

$|\vec{a} - \vec{b}| = \sqrt{(-20)^2 + 20^2}$

$= 20\sqrt{2}$

6a) $2(-2, 3) + (2, 1)$

$= (-4, 6) + (2, 1)$

$= (-2, 7)$

b) $-3(4, -9) - 9(2, 3)$

$= (-12, 27) + (-18, -27)$

$= (-30, 0)$

c) $-\frac{1}{2}(6, -2) + \frac{2}{3}(6, 15)$

$= (-3, 1) + (4, 10)$

$= (1, 11)$

7. $\vec{x} = 2\vec{i} - \vec{j}$ $\vec{y} = -\vec{i} + 5\vec{j}$

a) $3\vec{x} - \vec{y}$

$= 3(2\vec{i} - \vec{j}) - (-\vec{i} + 5\vec{j})$

$= 6\vec{i} - 3\vec{j} + \vec{i} - 5\vec{j}$

$= 7\vec{i} - 8\vec{j}$

b) $-(\vec{x} + 2\vec{y}) + 3(-\vec{x} - 3\vec{y})$

$= -\vec{x} - 2\vec{y} - 3\vec{x} - 9\vec{y}$

$= -4\vec{x} - 11\vec{y}$

$= -4(2\vec{i} - \vec{j}) - 11(-\vec{i} + 5\vec{j})$

$= -8\vec{i} + 4\vec{j} + 11\vec{i} - 55\vec{j}$

$= 3\vec{i} - 51\vec{j}$

7c) $2(\vec{x} + 3\vec{y}) - 3(\vec{y} + 5\vec{x})$

$= 2\vec{x} + 6\vec{y} - 3\vec{y} - 15\vec{x}$

$= -13\vec{x} + 3\vec{y}$

$= -13(2\vec{i} - \vec{j}) + 3(-\vec{i} + 5\vec{j})$

$= -26\vec{i} + 13\vec{j} - 3\vec{i} + 15\vec{j}$

$= -29\vec{i} + 28\vec{j}$

8a) $|\vec{x} + \vec{y}|$

$= |2\vec{i} - \vec{j} + (-\vec{i} + 5\vec{j})|$

$= |\vec{i} + 4\vec{j}|$

$= \sqrt{1^2 + 4^2}$

$= \sqrt{17}$

8b) $|\vec{x} - \vec{y}|$

$= |2\vec{i} - \vec{j} - (-\vec{i} + 5\vec{j})|$

$= |3\vec{i} - 6\vec{j}|$

$= \sqrt{3^2 + (-6)^2}$

$= \sqrt{45}$ or $3\sqrt{5}$

c) $|2\vec{x} - 3\vec{y}|$

$= |2(2\vec{i} - \vec{j}) - 3(-\vec{i} + 5\vec{j})|$

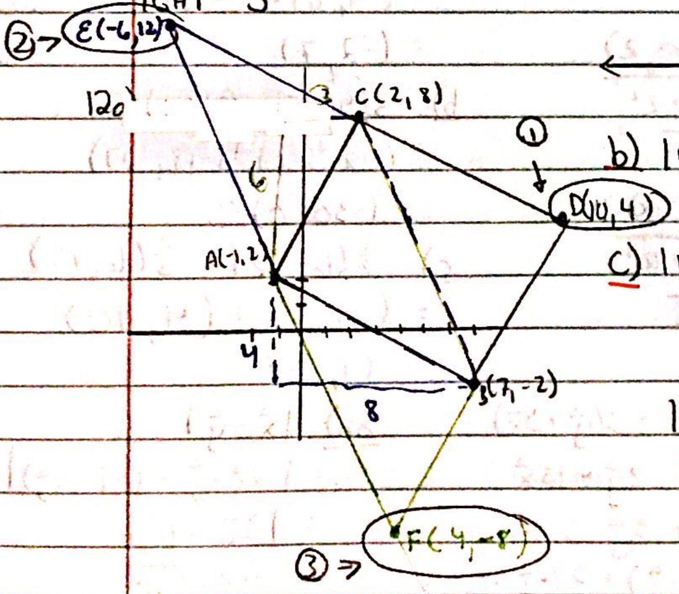
$= |7\vec{i} - 17\vec{j}|$

$= \sqrt{7^2 + 17^2}$

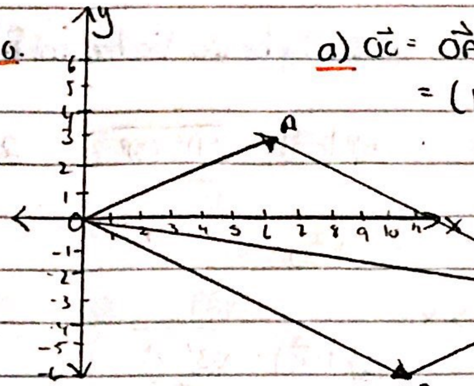
$= \sqrt{338}$

d) $|3\vec{y} - 2\vec{x}|$
$= |3(-\vec{i} + 5\vec{j}) - 2(2\vec{i} - \vec{j})|$
$= |-7\vec{i} + 17\vec{j}|$
$= \sqrt{338}$

9. $\vec{AB} = (4, 2)$
$|\vec{AB}| = \sqrt{4^2 + 2^2}$
$= 2\sqrt{5}$
$\vec{CD} = (2, 4)$
$|\vec{CD}| = \sqrt{2^2 + 4^2}$
$= 2\sqrt{5}$
$\vec{EF} = (-6, 4)$
$|\vec{EF}| = \sqrt{6^2 + 4^2}$
$= 2\sqrt{13}$
$\vec{GH} = (5, 0)$
$|\vec{GH}| = 5$



10.



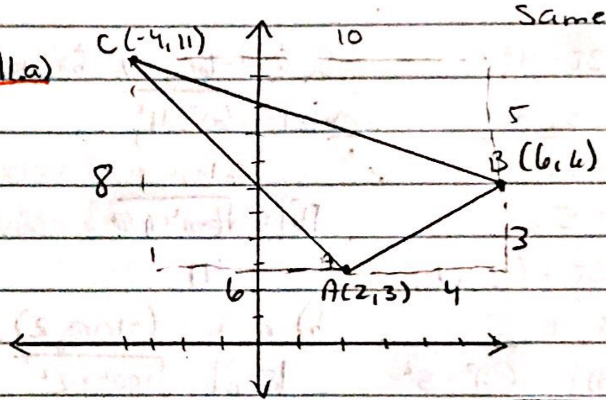
a) $\vec{OC} = \vec{OA} + \vec{OB}$
$= (17, -3)$

$\vec{BA} = (6 - 11, 3 + 6)$
$= (-5, 9)$

$\vec{BC} = (17 - 11, -3 + 6)$
$= (6, 3)$

b) $|\vec{OA}| = \sqrt{6^2 + 3^2} = |\vec{BC}|$ The two vectors have the same components, so same magnitude.
$= \sqrt{44}$ or $2\sqrt{11}$

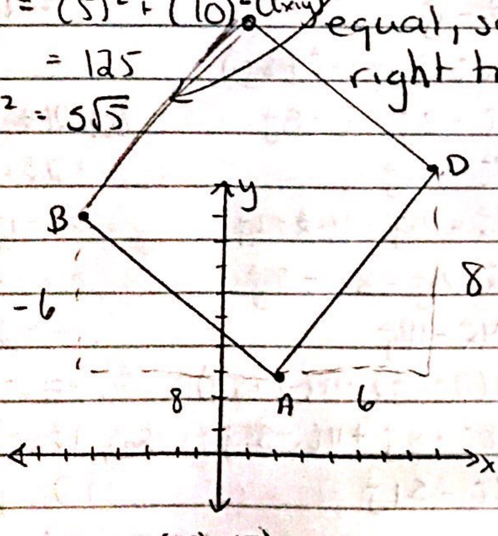
11 a)



b) $|\vec{AB}| = \sqrt{4^2 + 3^2}$ $= 5$
$|\vec{BC}| = \sqrt{10^2 + 5^2}$ $= 5\sqrt{5}$
$|\vec{AC}| = \sqrt{8^2 + 6^2}$ $= 10$

c) $|\vec{BC}|^2 = |\vec{AB}|^2 + |\vec{AC}|^2$
$= (5)^2 + (10)^2$
$= 125$
$|\vec{BC}|^2 = 5\sqrt{5}$
equal, so it is a right triangle.

14.

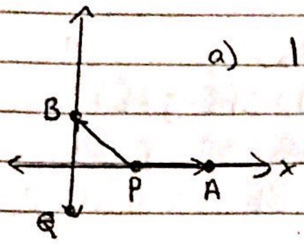


C(0, 17)

b) Move up 8 and right 6 from B.

13. $3(x, 1) - 5(2, 3y) = (11, 33)$
$(3x - 10, 3 - 15y) = (11, 33)$
$\therefore 3x - 10 = 11$ $\quad$ $3 - 15y = 33$
$3x = 21$ $\quad$ $-15y = 30$
$x = 7$ $\quad$ $y = -2$

15.



a) $|\vec{PA}| = \sqrt{a^2+4}$

$|\vec{PB}| = \sqrt{(5-a)^2+0}$

$= 5-a$

$|\vec{PA}| = |\vec{PB}|$

$\sqrt{a^2+4} = 5-a$

$a^2+4 = 25-10a+a^2$

$-21 = -10a$

$21/10 = a$ $\quad P(21/10, 0)$

b) Let $Q = (0, b)$

$|\vec{AQ}| = (2-b)$

$|\vec{BQ}| = \sqrt{25+b^2}$

$|\vec{AQ}| = |\vec{BQ}|$

$\sqrt{25+b^2} = 2-b$

$25+b^2 = 4-4b+b^2$ $\quad Q(0, -21/4)$

$21 = -4b$

$-21/4 = b$

6.7 Operations with Vectors in $\mathbb{R}^3$

1a) $\vec{OA} = -\vec{i}+2\vec{j}+4\vec{k}$

b) $|\vec{OA}| = \sqrt{(-1)^2+2^2+4^2}$

$= \sqrt{21}$

2. $\vec{OB} = (3,4,-4)$

$|\vec{OB}| = \sqrt{3^2+4^2+(-4)^2}$

$= \sqrt{41}$

3. $|\vec{a}+\frac{1}{3}\vec{b}-\vec{c}|$

$= |(1,3,-3)+\frac{1}{3}(-3,6,12)-(0,8,1)|$

$= |(1,3,-3)+(-1,2,4)-(0,8,1)|$

$= |(0,-3,0)|$

$= 3$

4a) $\vec{OP} = (-3,4,12)+(2,2,-1)$

$= (-1,6,11)$

b) $|\vec{OA}| = \sqrt{3^2+4^2+12^2}$ $\quad |\vec{OB}| = \sqrt{2^2+2^2+1^2}$

$= 13$ $\quad = 3$

$|\vec{OP}| = \sqrt{1^2+6^2+11^2}$

$= \sqrt{158}$

c) $\vec{AB} = (5,-2,-13)$

$|\vec{AB}| = \sqrt{5^2+2^2+13^2}$

$= \sqrt{198}$

$\vec{AB}$ is the vector from A to B

5a) $\vec{x}-2\vec{y}-\vec{z}$

$= (1,4,-1)-2(1,3,-2)-(-2,1,0)$

$= (1,4,-1)+(-2,-6,4)+(2,-1,0)$

$= (1,-3,3)$

b) $-2(1,4,-1)-3(1,3,-2)+(-2,1,0)$

$= (-2,-8,2)+(-3,-9,6)+(-2,1,0)$

$= (-7,-16,8)$

c) $\frac{1}{2}\vec{x}-\vec{y}+3\vec{z}$

$= \frac{1}{2}(1,4,-1)-(1,3,-2)+3(-2,1,0)$

$= (\frac{1}{2},2,-\frac{1}{2})+(-1,-3,2)+(-6,3,0)$

$= (-\frac{13}{2}, 2, \frac{3}{2})$

d) $3\vec{x}+5\vec{y}+3\vec{z}$

$= 3(1,4,-1)+5(1,3,-2)+3(-2,1,0)$

$= (3,12,-3)+(5,15,-10)+(-6,3,0)$

$= (2,30,-13)$

6a) $\vec{p}+\vec{q}$

$= 2\vec{i}-\vec{j}+\vec{k}+(-\vec{i}-\vec{j}+\vec{k})$

$= \vec{i}-2\vec{j}+2\vec{k}$

b) $\vec{p}-\vec{q}$

$= 2\vec{i}-\vec{j}+\vec{k}-(-\vec{i}-\vec{j}+\vec{k})$

$= 3\vec{i}+0\vec{j}+0\vec{k}$

c) $2\vec{p} - 5\vec{q}$
$= 2(2\vec{i} - \vec{j} + \vec{k}) - 5(-\vec{i} - \vec{j} + \vec{k})$
$= 4\vec{i} - 2\vec{j} + 2\vec{k} + 5\vec{i} + 5\vec{j} - 5\vec{k}$
$= 9\vec{i} + 3\vec{j} - 3\vec{k}$

d) $-2\vec{p} + 5\vec{q}$
$= -2(2\vec{i} - \vec{j} + \vec{k}) + 5(-\vec{i} - \vec{j} + \vec{k})$
$= -4\vec{i} + 2\vec{j} - 2\vec{k} - 5\vec{i} - 5\vec{j} + 5\vec{k}$
$= -9\vec{i} - 3\vec{j} + 3\vec{k}$

7a) $|\vec{m} - \vec{n}|$
$= |2\vec{i} - \vec{k} - (-2\vec{i} + \vec{j} + 2\vec{k})|$
$= |4\vec{i} - \vec{j} - 3\vec{k}|$
$= \sqrt{4^2 + 1^2 + 3^2}$
$= \sqrt{26}$

b) $|\vec{m} + \vec{n}|$
$= |2\vec{i} - \vec{k} + (-2\vec{i} + \vec{j} + 2\vec{k})|$
$= |\vec{j} + \vec{k}|$
$= \sqrt{1^2 + 1^2}$
$= \sqrt{2}$

c) $|2\vec{m} + 3\vec{n}|$
$= |2(2\vec{i} - \vec{k}) + 3(-2\vec{i} + \vec{j} + 2\vec{k})|$
$= |4\vec{i} - 2\vec{k} - 6\vec{i} + 3\vec{j} + 6\vec{k}|$
$= |-2\vec{i} + 3\vec{j} + 4\vec{k}|$
$= \sqrt{2^2 + 3^2 + 4^2}$
$= \sqrt{29}$

d) $|-5\vec{m}|$
$= |-5(2\vec{i} - \vec{k})|$
$= |-10\vec{i} + 5\vec{k}|$
$= \sqrt{10^2 + 5^2}$
$= 5\sqrt{5}$

8. $\vec{x} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$\oplus\ \vec{x} - \vec{y} = 3\vec{i} + 6\vec{j} - 7\vec{k}$
$2\vec{x} = 2\vec{i} + 8\vec{j} - 2\vec{k}$
$\boxed{\vec{x} = \vec{i} + 4\vec{j} - \vec{k}}$

$\vec{x} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$\vec{i} + 4\vec{j} - \vec{k} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$\vec{y} = -2\vec{i} - 2\vec{j} + 6\vec{k}$

9a) $\vec{OA} = (a, b, 0)$ represents any vector that lies in the xy-plane.
$\vec{OB} = (a, 0, c)$ " " " " " " " xz-plane.
$\vec{OC} = (0, b, c)$ " " " " " " yz-plane.
b) $\vec{OA} = a\vec{i} + b\vec{j} + 0\vec{k}$, $\vec{OB} = a\vec{i} + 0\vec{j} + c\vec{k}$, $\vec{OC} = 0\vec{i} + b\vec{j} + c\vec{k}$
c) $|\vec{OA}| = \sqrt{a^2 + b^2}$ $|\vec{OB}| = \sqrt{a^2 + c^2}$ $|\vec{OC}| = \sqrt{b^2 + c^2}$
d) $\vec{AB} = (0, -b, c)$, direction vector from A to B (on the yz plane)

10. $A(-2, -6, 3)$ $B(3, -4, 12)$
a) $|\vec{OA}| = \sqrt{2^2 + 6^2 + 3^2}$
$= 7$
b) $|\vec{OB}| = \sqrt{3^2 + 4^2 + 12^2}$
$= 13$
c) $\vec{AB} = (5, 2, 9)$
d) $|\vec{AB}| = \sqrt{5^2 + 2^2 + 9^2}$
$= \sqrt{110}$
e) $\vec{BA} = (-5, -2, -9)$
f) $|\vec{BA}| = \sqrt{110}$

11. $\vec{AB} = (3, -4, 12)$ $\vec{BC} = (4, -2, -2)$ $\vec{CD} = (-3, 4, -12)$
$\vec{DA} = (-4, 2, 2)$
$|\vec{AB}| = |\vec{CD}| = 13$ $|\vec{BC}| = |\vec{DA}| = 2\sqrt{6}$
$\therefore$ This has to be a parallelogram (two pairs of equal opposite sides)
* you can also prove this with direction of vectors (see back of book)

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12. $2\vec{x}+\vec{y}-2\vec{z}=\vec{0}$ $\quad \vec{x}=(-1,b,c),\ \vec{y}=(a,-2,c),\ \vec{z}=(-a,6,c)$

$= 2(-1,b,c)+(a,-2,c)-2(-a,6,c)=\vec{0}$

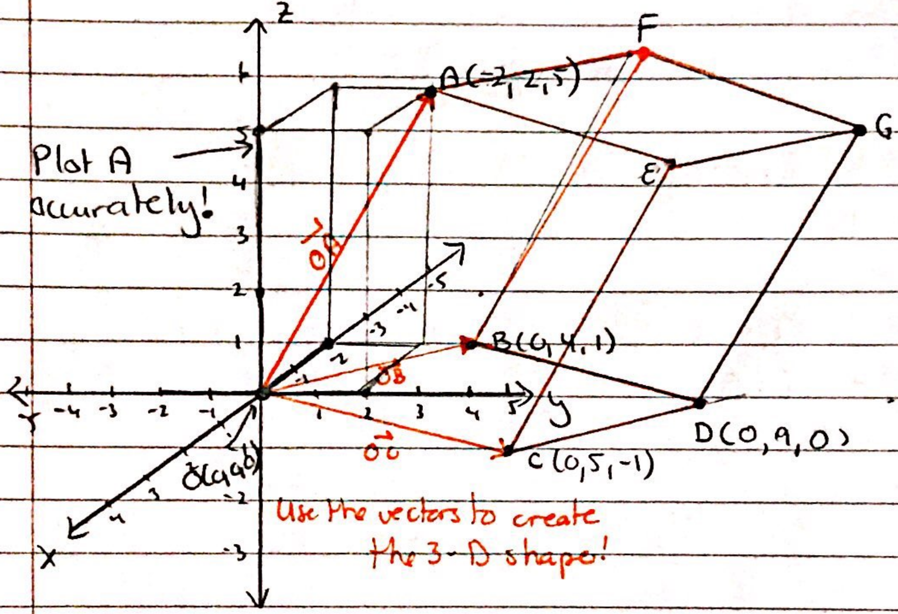
$(-2,2b,2c)+(a,-2,c)+(2a,-12,-2c)=(0,0,0)$

$(3a-2,\ 2b-14,\ c)=(0,0,0)$

$\therefore 3a-2=0 \qquad 2b-14=0 \qquad c=0$

$a=2/3 \qquad b=7$

13. ☆ Parallelepiped → 3-D parallelogram, made of 6 parallelograms. ☆



D(0, 9, 0)
E(-2, 7, 4)
F(-2, 6, 6)
G(-2, 11, 5)

14. Let C(a, 0, 0)

$|\vec{AC}|=|\vec{BC}|$

$|(a+2,-1,-3)|=|(a-4,1,3)|$

$\left(\sqrt{(a+2)^2+1^2+3^2}\right)^2=\left(\sqrt{(a-4)^2+1^2+3^2}\right)^2$

$(a+2)^2+10=(a-4)^2+10$

$(a+2)^2=(a-4)^2$

$a^2+4a+4=a^2-8a+16$

$12a=12$

$a=1$

C(1, 0, 0)